1.) $\begin{cases} a u_{tt} = b u_{xx} - c u_t \\ u(0,t) = u(L,t) = 0 \\ u(x,0) = f(x), \quad u_t(x,0) = g(x) \end{cases}$ $\qquad c^2 < \dfrac{4\pi^2 a b}{L^2}$

suppose $u(x,t) = h(x)\,g(t)$ so plugging into PDE $\Rightarrow a h(x) \dfrac{d^2 g}{dt^2} = b g(t) \dfrac{d^2 h}{dx^2} - c h(x) \dfrac{dg}{dt}$

$\Rightarrow h(x)\left(a \dfrac{d^2 g}{dt^2} + c \dfrac{dg}{dt}\right) = b g(t) \dfrac{d^2 h}{dx^2} \Rightarrow \underbrace{\dfrac{1}{b}\dfrac{1}{g}\left(a \dfrac{d^2 g}{dt^2} + c\dfrac{dg}{dt}\right)}_{t} = \underbrace{\dfrac{1}{h}\dfrac{d^2 h}{dx^2}}_{x} = -\lambda$

1st: $a \dfrac{d^2 g}{dt^2} + c \dfrac{dg}{dt} + b\lambda g = 0$ $\qquad$ 2nd: $\begin{cases} \dfrac{d^2 h}{dx^2} + \lambda h = 0 \\ h(0) = h(L) = 0 \end{cases}$

for 2nd: $r^2 + \lambda = 0 \quad r = \pm\sqrt{-\lambda}$ $\qquad$ case $\underline{\lambda = 0}$: $h(x) = c_1 + c_2 x$

$0 = h(0) = c_1 \Rightarrow h(x) = c_2 x$ and $0 = h(L) = c_2 L \Rightarrow c_2 = 0$

case $\underline{\lambda < 0}$: case $-\lambda = s^2 \Rightarrow r = \pm s$ so $h(x) = c_1 e^{sx} + c_2 e^{-sx}$

$0 = h(0) = c_1 + c_2 \Rightarrow c_1 = -c_2 \Rightarrow h(x) = c_1(e^{sx} - e^{-sx})$ so $0 = h(L) = c_1(e^{sL} - e^{-sL})$

$\Rightarrow e^{sL} = e^{-sL} \Rightarrow s = 0$ but $s \neq 0$ contradiction so $c_1 = 0$ and $c_2 = 0$

case $\underline{\lambda > 0}$: $\Rightarrow h(x) = c_1 \cos(\sqrt{\lambda} x) + c_2 \sin(\sqrt{\lambda} x)$ and $0 = h(0) = c_1$ so $h(x) = c_2 \sin(\sqrt{\lambda} x)$

$0 = h(L) = c_2 \sin(\sqrt{\lambda} L) \Rightarrow \sqrt{\lambda} L = n\pi \Rightarrow \lambda_n = \left(\dfrac{n\pi}{L}\right)^2$ so $h_n(x) = \sin\left(\dfrac{n\pi x}{L}\right)$

1st: $a \dfrac{d^2 g}{dt^2} + c\dfrac{dg}{dt} + b\lambda g = 0 \Rightarrow a r^2 + c r + b\lambda = 0$ so $r = \dfrac{-c \pm \sqrt{c^2 - 4ab\lambda}}{2a}$

since $c^2 < \dfrac{4\pi^2 a b}{L^2} \Rightarrow c^2 - \dfrac{4\pi^2 a b}{L^2} < 0$ but smallest $\lambda_n = \left(\dfrac{\pi}{L}\right)^2$ so for all λ_n $c^2 - 4ab\lambda_n < 0$

so set $c^2 - 4ab\lambda_n = -\varepsilon_n^2 \Rightarrow r = \dfrac{-c \pm i\varepsilon_n}{2a} = -d \pm i\gamma_n$ w/ $d = \dfrac{c}{2a}$, $\gamma_n = \dfrac{\varepsilon_n}{2a}$

so $g(t) = c_1 e^{-dt}\cos(\gamma_n t) + c_2 e^{-dt}\sin(\gamma_n t)$

so $u(x,t) = \displaystyle\sum_{n=1}^{\infty} A_n e^{-dt}\cos(\gamma_n t)\sin\left(\dfrac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n e^{-dt}\sin(\gamma_n t)\sin\left(\dfrac{n\pi x}{L}\right)$

$A_n = \dfrac{2}{L}\displaystyle\int_0^L f(x)\sin\left(\dfrac{n\pi x}{L}\right)dx, \quad B_n = \dfrac{1}{\gamma_n}\dfrac{2}{L}\int_0^L \left(g(x) + d f(x)\right)\sin\left(\dfrac{n\pi x}{L}\right)dx$

2) $\begin{cases} a u_{tt} = b u_{xx} - c u_t \\ u(0,t) = u(L,t) = 0 \\ u(x,0) = f(x), \quad u_t(x,0) = g(x) \end{cases}$ $\qquad c^2 > \frac{n^2\pi^2 ab}{L^2}$

S'pose $u(x,t) = h(x) g(t)$, plugging into PDE $\Rightarrow a h \frac{d^2 g}{dt^2} = b g \frac{d^2 h}{dx^2} - c h \frac{dg}{dt}$

$\Rightarrow \frac{1}{b}\frac{1}{g}\left(a \frac{d^2 g}{dt^2} + c \frac{dg}{dt}\right) = \frac{1}{h}\frac{d^2 h}{dx^2} = -\lambda \Rightarrow$ 1st: $a \frac{d^2 g}{dt^2} + c \frac{dg}{dt} + b\lambda g = 0$ $\qquad$ 2nd: $\begin{cases} \frac{d^2 h}{dx^2} + \lambda h = 0 \\ h(0) = h(L) = 0 \end{cases}$

for 2nd: $r^2 + \lambda = 0$ $\quad r = \pm\sqrt{-\lambda}$ $\quad$ case: $\lambda = 0$ $\quad h(x) = c_1 + c_2 x$ $\quad 0 = h(0) = c_1 \Rightarrow h(x) = c_2 x$

so $0 = h(L) = c_2 L \Rightarrow c_2 = 0$ $\quad$ case: $\lambda < 0$ $\quad$ let $-\lambda = s^2$, $\Rightarrow r = \pm s$ so $h(x) = c_1 e^{sx} + c_2 e^{-sx}$

$0 = h(0) = c_1 + c_2 \Rightarrow c_2 = -c_1$ so $h(x) = c_1(e^{sx} - e^{-sx})$ so $0 = h(L) = c_1(e^{sL} - e^{-sL})$

$\Rightarrow e^{-sL} = e^{sL} \Rightarrow s = 0$ but $s \neq 0$ contradiction so $c_1 = 0 \Rightarrow c_2 = 0$

case $\lambda > 0$: so $h(x) = c_1 \cos(\sqrt{\lambda} x) + c_2 \sin(\sqrt{\lambda} x)$ & $0 = h(0) = c_1 \Rightarrow h(x) = c_2 \sin(\sqrt{\lambda} x)$

so $0 = h(L) = c_2 \sin(\sqrt{\lambda} L) \Rightarrow \sqrt{\lambda} L = n\pi \Rightarrow \lambda_n = \left(\frac{n\pi}{L}\right)^2$ so $h_n(x) = \sin\left(\frac{n\pi x}{L}\right)$

for 1st: $a \frac{d^2 g}{dt^2} + c \frac{dg}{dt} + b\lambda_n g = 0 \Rightarrow a r^2 + c r + b\lambda = 0$ so $r = \frac{-c \pm \sqrt{c^2 - 4ab\lambda}}{2a}$

but $c^2 > \frac{n^2\pi^2 ab}{L^2}$ for all $n \Rightarrow c^2 - \left(\frac{n\pi}{L}\right)^2 ab > 0 \Rightarrow c^2 - 4ab\lambda_n > 0$ for all n

set $c^2 - 4ab\lambda_n = \epsilon_n^2 \Rightarrow r = \frac{-c \pm \epsilon_n}{2a} = -d \pm \gamma_n$ $\quad d = \frac{c}{2a}$, $\quad \gamma_n = \frac{\epsilon_n}{2a}$

so $g(t) = c_1 e^{-dt} e^{\gamma_n t} + c_2 e^{-dt} e^{-\gamma_n t} = c_1 e^{(-d+\gamma_n)t} + c_2 e^{-(d+\gamma_n)t}$

so $u(x,t) = \sum_{n=1}^{\infty} A_n e^{(-d+\gamma_n)t} \sin\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n e^{-(d+\gamma_n)t} \sin\left(\frac{n\pi x}{L}\right)$

so $A_n = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$ and $B_n = \frac{2}{L}\int_0^L \left(\frac{-d+\gamma_n}{d+\gamma_n} f(x) - \frac{1}{d+\gamma_n} g(x)\right) \sin\left(\frac{n\pi x}{L}\right) dx$

3.) $\begin{cases} a(x)b(x)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(c(x)\frac{\partial u}{\partial x}\right) + \alpha(x)u \\ u_x(0,t) = u_x(L,t) = 0 \\ u(x,0) = f(x) \end{cases}$

s'pose $u(x,t) = h(x)g(t)$ plugging into PDE $\Rightarrow$ $a(x)b(x)h(x)\frac{dg}{dt} = \frac{d}{dx}\left(c(x)g(t)\frac{dh}{dx}\right) + \alpha(x)h(x)g(t)$

$\Rightarrow$ $abh\frac{dg}{dt} = g\left(\frac{d}{dx}\left(c\frac{dh}{dx}\right) + \alpha h\right)$ $\Rightarrow$ 1st: $\frac{dg}{dt} = -\lambda g$ 2nd: $\begin{cases} \frac{d}{dx}\left(c\frac{dh}{dx}\right) + \alpha h + ab\lambda h = 0 \\ \frac{dh}{dx}(0) = \frac{dh}{dx}(L) = 0 \end{cases}$

1st: $g(t) = c_1 e^{-\lambda t}$, the 2nd $\begin{cases} \frac{d}{dx}\left(c\frac{dh}{dx}\right) + \alpha h + ab\lambda h = 0 \\ \frac{dh}{dx}(0) = \frac{dh}{dx}(L) = 0 \end{cases}$ is a S-L type problem w/ Neumann B.C. and weight $\sigma = ab$

so by the S-L Thm we can solve the eq. for each eigenvalue $\lambda_n \longleftrightarrow \varphi_n(x)$ eigenfuncs.

so $u(x,t) = \sum_{n=0}^{\infty} A_n e^{-\lambda_n t}\varphi_n(x)$ and $A_n = \frac{\int_0^L f(x)\varphi_n(x)ab\,dx}{\int_0^L \varphi_n^2(x)ab\,dx}$ as $t \to \infty$ $u(x,t) \to A_0$ since $\lambda \geq 0$ by R.Q.

4.) $\begin{cases} a(x)b(x)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(c(x)\frac{\partial u}{\partial x}\right) + \alpha(x)u \\ u(-L,t) = u(L,t), \quad u_x(-L,t) = u_x(L,t) \\ u(x,0) = f(x) \end{cases}$

s'pose $u(x,t) = h(x)g(t)$ plugging into PDE $\Rightarrow$ $abh\frac{dg}{dt} = \frac{d}{dx}\left(cg\frac{dh}{dx}\right) + \alpha h g$

$\Rightarrow$ $abh\frac{dg}{dt} = g\left(\frac{d}{dx}\left(c\frac{dh}{dx}\right) + \alpha h\right)$ $\Rightarrow$ 1st: $\frac{dg}{dt} = -\lambda g$, 2nd: $\begin{cases} \frac{d}{dx}\left(c\frac{dh}{dx}\right) + \alpha h + ab\lambda h = 0 \\ h(-L) = h(L), \quad \frac{dh}{dx}(-L) = \frac{dh}{dx}(L) \end{cases}$

1st: $g(t) = c_1 e^{-\lambda t}$ then 2nd: $\begin{cases} \frac{d}{dx}\left(c\frac{dh}{dx}\right) + \alpha h + ab\lambda h = 0 \\ h(-L) = h(L), \quad \frac{dh}{dx}(-L) = \frac{dh}{dx}(L) \end{cases}$ is a S-L type problem w/ periodic B.C. and weight $\sigma = ab$

so by the S-L Thm we can solve the eq. at each eigenvalue $\lambda_n \longleftrightarrow \varphi_n(x)$ eigenfuncs.

so $u(x,t) = \sum_{n=0}^{\infty} A_n e^{-\lambda_n t}\varphi_n(x)$ and $A_n = \frac{\int_{-L}^{L} f(x)\varphi_n(x)ab\,dx}{\int_{-L}^{L} \varphi_n^2(x)ab\,dx}$

as $t \to \infty$ $u(x,t) \longrightarrow A_0$

since $\lambda \geq 0$ by R.Q.

5.) $\begin{cases} u_t = k\Delta u \\ u(0,y,t) = u(L,y,t) = 0 \\ u(x,0,t) = u(x,H,t) = 0 \\ u(x,y,0) = \alpha(x,y) \end{cases}$

s'pose $u(x,y,t) = h(t)\,v(x,y)$ plugging into PDE $\Rightarrow \frac{dh}{dt} v = k h \Delta v \Rightarrow \frac{1}{k}\frac{1}{h}\frac{dh}{dt} = \frac{1}{v}\Delta v = -\lambda$

$\Rightarrow$ 1st: $\frac{dh}{dt} = -k\lambda h$ and 2nd: $\begin{cases} \Delta v + \lambda v = 0 \\ v(0,y) = v(L,y) = 0 \\ v(x,0) = v(x,H) = 0 \end{cases}$

1st: $h(t) = ce^{-k\lambda t}$ for 2nd: s'pose $v(x,y) = f(x)g(y) \Rightarrow \frac{d^2 f}{dx^2} g + f\frac{d^2 g}{dy^2} = -\lambda f g$

$\Rightarrow \frac{1}{f}\frac{d^2 f}{dx^2} + \frac{1}{g}\frac{d^2 g}{dy^2} = -\lambda \Rightarrow \underbrace{\frac{1}{f}\frac{d^2 f}{dx^2}}_{x} = \underbrace{-\lambda - \frac{1}{g}\frac{d^2 g}{dy^2}}_{y} = -\mu$

$\Rightarrow$ 1st: $\begin{cases} \frac{d^2 f}{dx^2} + \mu f = 0 \\ f(0) = f(L) = 0 \end{cases}$ and 2nd: $\begin{cases} \frac{d^2 g}{dy^2} + (\lambda - \mu) g = 0 \\ g(0) = g(H) = 0 \end{cases}$

for 1st: we know for Dirichlet B.C. we set $\mu_n = \left(\frac{n\pi}{L}\right)^2$ and $f_n(x) = \sin\left(\frac{n\pi x}{L}\right)$ $n \geq 1$

in 2nd: each λ is dependent on μ_n so λ_{mn} has multi-index again we have Dirichlet B.C. so we know $\lambda_{mn} - \mu_n = \left(\frac{m\pi}{H}\right)^2 \Rightarrow \lambda_{mn} = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{H}\right)^2$

and so $g_m(y) = \sin\left(\frac{m\pi y}{H}\right)$, $m \geq 1$ thus the eigenfuncs to 2nd: is $v_{mn}(x,y) = \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{H}\right)$

so $u(x,y,t) = \sum_{m,n=1}^{\infty} A_{mn} e^{-k\lambda_{mn} t} \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{H}\right)$

and $A_{mn} = \frac{4}{LH}\int_0^L\int_0^H \alpha(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{H}\right) dy\,dx$

6) $u_t = \kappa \Delta u$

$$\begin{cases} u_x(0,y,t) = u_x(L,y,t) = 0 \\ u(x,0,t) = u(x,H,t) = 0 \\ u(x,y) = \alpha(x,y) \end{cases}$$

s'pose $u(x,y,t) = h(t)\,v(x,y)$ plugging into PDE $\Rightarrow \frac{dh}{dt} v = \kappa h \Delta v \Rightarrow \frac{1}{\kappa}\frac{1}{h}\frac{dh}{dt} = \frac{1}{v}\Delta v = -\lambda$

$\Rightarrow$ 1st: $\frac{dh}{dt} = -\kappa\lambda h$ & 2nd: $\begin{cases} \Delta v + \lambda v = 0 \\ v_x(0,y) = v_x(L,y) = 0 \\ v(x,0) = v(x,H) = 0 \end{cases}$

1st: get $h(t) = c_1 e^{-\kappa\lambda t}$ for 2nd: s'pose $v(x,y) = f(x)g(y) \Rightarrow \frac{d^2f}{dx^2}g + f\frac{d^2g}{dy^2} = -\lambda fg$

$\Rightarrow \frac{1}{f}\frac{d^2f}{dx^2} + \frac{1}{g}\frac{d^2g}{dy^2} = -\lambda \Rightarrow \underbrace{\frac{1}{f}\frac{d^2f}{dx^2}}_{x} = \underbrace{-\lambda - \frac{1}{g}\frac{d^2g}{dy^2}}_{y} = -\mu$

$\Rightarrow$ 1st*: $\begin{cases} \frac{d^2f}{dx^2} + \mu f = 0 \\ \frac{df}{dx}(0) = \frac{df}{dx}(L) = 0 \end{cases}$ & 2nd*: $\begin{cases} \frac{d^2g}{dy^2} + (\lambda - \mu)g = 0 \\ g(0) = g(H) = 0 \end{cases}$

For 1st* we know for Neumann B.C. we get $\mu_n = \left(\frac{n\pi}{L}\right)^2$ and $f_n(x) = \cos\left(\frac{n\pi x}{L}\right)$ $n \geq 0$

in 2nd* each λ is dependent on μ_n so λ_{mn} has multi-index ~~again~~ here we have Dirichlet B.C. so we get $\lambda_{mn} - \mu_n = \left(\frac{m\pi}{H}\right)^2$ and $\lambda_{mn} = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{H}\right)^2$

and $g_m(y) = \sin\left(\frac{m\pi y}{H}\right)$ $m \geq 1$ Thus the eigenfncs to 2nd are $v_{mn}(x,y) = \sin\left(\frac{m\pi y}{H}\right)\cos\left(\frac{n\pi x}{L}\right)$

so $u(x,y,t) = \sum_{\substack{n=0\\m=1}}^{\infty} A_{mn} e^{-\kappa\lambda_{mn}t} \sin\left(\frac{m\pi y}{H}\right)\cos\left(\frac{n\pi x}{L}\right)$

and $A_{mn} = \frac{4}{LH}\int_0^H\int_0^L \alpha(x,y)\sin\left(\frac{m\pi y}{H}\right)\cos\left(\frac{n\pi x}{L}\right)dx\,dy$ for $m \geq 1, n \geq 1$

and $A_{m0} = \frac{2}{LH}\int_0^H\int_0^L \alpha(x,y)\sin\left(\frac{m\pi y}{H}\right)dx\,dy$ for $m \geq 1$

7.) $\begin{cases} L(h)+\lambda\sigma h=0 \\ h(a)=h(b)=0 \end{cases}$, $L(h)=\frac{d}{dx}\left(p\frac{dh}{dx}\right)+qh$

s'pose λ_n,λ_m are different eigenvalues i.e. $m\neq n$ then we get by S-L Thm

$\lambda_n \leftrightarrow \varphi_n(x)$, $\lambda_m \leftrightarrow \varphi_m(x)$ eigenfncs. Then we get $\begin{cases} L(\varphi_n) = -\lambda_n\sigma\varphi_n \\ L(\varphi_m) = -\lambda_m\sigma\varphi_m \end{cases}$

$\Rightarrow \begin{cases} \varphi_m L(\varphi_n) = -\lambda_n\sigma\varphi_n\varphi_m \\ \varphi_n L(\varphi_m) = -\lambda_m\sigma\varphi_m\varphi_n \end{cases} \Rightarrow (\lambda_m-\lambda_n)\sigma\varphi_n\varphi_m = \varphi_m L(\varphi_n) - \varphi_n L(\varphi_m)$

$$\Rightarrow (\lambda_m-\lambda_n)\int_a^b \varphi_n\varphi_m\sigma\,dx = \int_a^b\left(\varphi_m L(\varphi_n) - \varphi_n L(\varphi_m)\right)dx \overset{\text{Lagrange Id.}}{=} p\left(\varphi_m\frac{d\varphi_n}{dx} - \varphi_n\frac{d\varphi_m}{dx}\right)\Big|_a^b$$

$$= p\left(\varphi_m(b)\frac{d\varphi_n}{dx}(b) - \varphi_n(b)\frac{d\varphi_m}{dx}(b) - \varphi_m(a)\frac{d\varphi_n}{dx}(a) + \varphi_n(a)\frac{d\varphi_m}{dx}(a)\right) = 0 \quad \text{by B.C.}$$

$\Rightarrow (\lambda_m-\lambda_n)\int_a^b \varphi_n\varphi_m\sigma\,dx = 0$ but $\lambda_n\neq\lambda_m$ as they are different eigenvalues

$\Rightarrow \int_a^b \varphi_n\varphi_m\sigma\,dx=0$ thus φ_n,φ_m orthogonal for $n\neq m$ $\square$

8.) $\begin{cases} L(h)+\lambda\sigma h=0 \\ h(a)=h(b)=0 \end{cases}$, $L(h)=\frac{d}{dx}\left(p\frac{dh}{dx}\right)+qh$

s'pose λ eigenvalue then by S-L Thm, $\lambda \leftrightarrow \varphi(x)$ is an eigenfnc. so by since complex #'s come in pairs for $\bar\lambda \leftrightarrow \bar\varphi(x)$ eigenfnc. and so $\begin{cases} L(\varphi) = -\lambda\sigma\varphi \\ L(\bar\varphi) = -\bar\lambda\sigma\varphi \end{cases}$

$\Rightarrow \begin{cases} \bar\varphi L(\varphi) = -\lambda\sigma\varphi\bar\varphi \\ \varphi L(\bar\varphi) = -\bar\lambda\sigma\varphi\bar\varphi \end{cases} \Rightarrow (\lambda-\bar\lambda)\sigma|\varphi|^2 = \varphi L(\bar\varphi) - \bar\varphi L(\varphi)$

$$\Rightarrow (\lambda-\bar\lambda)\int_a^b |\varphi|^2\sigma\,dx = \int_a^b\left(\varphi L(\bar\varphi) - \bar\varphi L(\varphi)\right)dx \overset{\text{Lagrange Id.}}{=} p\left(\varphi\frac{d\bar\varphi}{dx} - \bar\varphi\frac{d\varphi}{dx}\right)\Big|_a^b$$

$$= p\left(\varphi(b)\frac{d\bar\varphi}{dx}(b) - \bar\varphi(b)\frac{d\varphi}{dx}(b) - \varphi(a)\frac{d\bar\varphi}{dx}(a) + \bar\varphi(a)\frac{d\varphi}{dx}(a)\right) = 0 \quad \text{by B.C.}$$

$\Rightarrow (\lambda-\bar\lambda)\int_a^b |\varphi|^2\sigma\,dx = 0 \Rightarrow \lambda-\bar\lambda=0$ or $\int_a^b |\varphi|^2\sigma\,dx=0$

but $\int_a^b |\varphi|^2\sigma\,dx>0$ since weight $\sigma>0$ and $|\varphi|^2>0$ $\Rightarrow \lambda=\bar\lambda$ thus λ is real $\square$

9.) $u(x,t) = \frac{1}{2}\left(g(x+t) - g(t-x) + \int_{t-x}^{x+t} h(y)\,dy\right)$

let $u = x+t$, $v = t-x$

so $u(x,t) = \frac{1}{2}\left(g(u) - g(v) + \int_v^u h(y)\,dy\right)$

th $u_t = \frac{1}{2}\left(\frac{dg}{du}\frac{\partial u}{\partial t} - \frac{dg}{dv}\frac{\partial v}{\partial t} + h(u)\frac{\partial u}{\partial t} - h(v)\frac{\partial v}{\partial t}\right) = \frac{1}{2}\left(\frac{dg}{du} - \frac{dg}{dv} + h(u) - h(v)\right)$

$u_{tt} = \frac{1}{2}\left(\frac{d^2g}{du^2}\frac{\partial u}{\partial t} - \frac{d^2g}{dv^2}\frac{\partial v}{\partial t} + \frac{dh}{du}\frac{\partial u}{\partial t} - \frac{dh}{dv}\frac{\partial v}{\partial t}\right) = \frac{1}{2}\left(\frac{d^2g}{du^2} - \frac{d^2g}{dv^2} + \frac{dh}{du} - \frac{dh}{dv}\right) = A$

OTH: $u_x = \frac{1}{2}\left(\frac{dg}{du}\frac{\partial u}{\partial x} - \frac{dg}{dv}\frac{\partial v}{\partial x} + h(u)\frac{\partial u}{\partial x} - h(v)\frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\frac{dg}{du} + \frac{dg}{dv} + h(u) + h(v)\right)$

$u_{xx} = \frac{1}{2}\left(\frac{d^2g}{du^2}\frac{\partial u}{\partial x} + \frac{d^2g}{dv^2}\frac{\partial v}{\partial x} + \frac{dh}{du}\frac{\partial u}{\partial x} + \frac{dh}{dv}\frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\frac{d^2g}{du^2} - \frac{d^2g}{dv^2} + \frac{dh}{du} - \frac{dh}{dv}\right) = B$

so $A = B$ thus $u_{tt} = u_{xx}$