Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Circle or box your answers. Show all work. Check your answers.

Name: ____________________

ID number: ____________________

Question	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	4	
10	4	
11	4	
12	4	
13	4	
14	4	
15	4	
16	6	
17	6	
18	6	
19	6	
20	6	
21	6	
22	6	
Total:	102	

1. (4 points) If $f(x) = x^2$, $g(x) = 3x - 6$, evaluate $(f - g)(x)$, $(fg)(3)$, and $\frac{f}{g}(2)$

Solution:

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= x^2 - (3x - 6)\\ &= \boxed{x^2 - 3x + 6}\\ (fg)(3) &= f(3)g(3)\\ &= 3^2(3(3) - 6)\\ &= 9(9 - 6)\\ &= 9(3)\\ &= \boxed{27}\\ (\frac{f}{g})(2) &= \frac{f(2)}{g(2)}\\ &= \frac{2^2}{3(2) - 6}\\ &= \frac{4}{6 - 6}\\ &= \frac{4}{0}\end{aligned}$$

There is a 0 on the denominator, so $\frac{f}{g}(2)$ $\boxed{\text{does not exist}}$.

2. (4 points) Determine if the function $h(x) = 3x^{17} - x$ is even, odd, or neither

Solution: I first calculate $f(-x)$:

$$\begin{aligned}f(-x) &= 3(-x)^{17} - (-x)\\ &= -3x^{17} + x\end{aligned}$$

This is not the original function, so the function is not even. Now I calculate $-f(x) = -3x^{17} - x$. This is $f(-x)$, so the function is $\boxed{\text{odd}}$.

3. (4 points) Determine the shifts that have occurred to $f(x)$ when

$$g(x) = -f(5x) + 17$$

Solution: The graph has been

$\boxed{\text{reflected over the x-axis, horizontally stretched by 5, and vertically shifted by 17}}$

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4. (4 points) Given the point $f(5) = 17$, find a point on the function $g(x) = 2f(x-3) - 5$

Solution: I find x when $x - 3 = 5$. So $x = 8$. Then

$$\begin{aligned} g(8) &= 2f(8-3) - 5 \\ &= 2f(5) - 5 \\ &= 2(17) - 5 \\ &= 34 - 5 \\ &= 29 \end{aligned}$$

So a point on the graph $g(x)$ is $\boxed{(8, 29)}$.

5. (4 points) Write a function $g(x)$ that has been moved to the left by 4 and reflected across the x-axis from the graph $f(x)$.

Solution: A function that moves to the left by 4, is $f(x+4)$. A function that is reflected across the x-axis is $-f(x)$. Then the function $g(x)$ is $\boxed{g(x) = -f(x+4)}$.

6. (4 points) Write the equation of a horizontal line that passes through the point $(0, 2)$. What is its slope?

Solution: A horizontal line has slope $\boxed{0}$. We are given a point, so I can write the equation of the line in point-slope form, as $y - 2 = 0$. I can write this in the more familiar form: $\boxed{y = 2}$.

7. (4 points) If $(7, k)$ and $(2, 4)$ have a slope of $\frac{1}{2}$, find k.

Solution: I use the slope formula with $x_1 = 7$, $y_1 = k$, $x_2 = 2$, and $y_2 = 4$. So

$$\begin{aligned} \frac{1}{2} &= \frac{4-k}{2-7} \\ \frac{1}{2} &= \frac{4-k}{-5} \\ -\frac{5}{2} &= 4 - k \\ -\frac{5}{2} - 4 &= -k \\ -\frac{5}{2} - \frac{8}{2} &= -k \\ -\frac{13}{2} &= -k \\ \frac{13}{2} &= k \end{aligned}$$

Therefore, $\boxed{\dfrac{13}{2}}$.

8. (4 points) Find the equation of the line in standard form with slope of $-\frac{4}{5}$ and y-intercept at $(0, 3)$.

Solution: I can first put the line in point-intercept form by $y = -\frac{4}{5}x + 3$. Now I convert to standard form by

$$0 = -y - \frac{4}{5}x + 3$$
$$0 = -5y - 4x + 15$$
$$0 = 5y + 4x - 15$$
$$\boxed{0 = 4x + 5y - 15}$$

9. (4 points) Find the equation of the line parallel to

$$3x + 10y = 2$$

and passes through $(8, 4)$

Solution: I put the given line equation in point-intercept form by

$$10y = -3x + 2$$
$$y = -\frac{3}{10}x + \frac{2}{10}$$

The slope of the line is then $\frac{3}{10}$. Therefore, the slope of the parallel line is also $\frac{3}{10}$. I can then make the new line in point-slope form by $\boxed{y - 4 = -\frac{3}{10}(x - 8)}$.

10. (4 points) Find the line perpendicular to the line

$$10x + 11y = 3$$

and passes through $(9, 7)$.

Solution: I first put the line given in slope-intercept form:

$$11y = 3 - 10x$$
$$y = -\frac{10}{11}x + \frac{3}{11}$$

The slope of this line is $-\frac{10}{11}$. Then the slope of the perpendicular line is $\frac{11}{10}$. I can then put my perpendicular line in point-slope form. This is $\boxed{y - 7 = \frac{11}{10}(x - 9)}$.

11. (4 points) Find the number of real roots of $3x^2 + 5x + 17$. You must show work to get full credit.

Solution: For this quadratic, $A = 3$, $B = 5$, and $C = 17$. Thus

$$\begin{aligned} B^2 - 4AC &= 5^2 - 4(3)(17) \\ &= 25 - 12(17) \\ &= 204 \end{aligned}$$

This is a positive number, so there are two real roots.

12. (4 points) Find the vertex of the parabola at $f(x) = -5x^2 + 20x - 16$. Is this point a maximum or minimum?

Solution: For this quadratic, $A = -5$, $B = 20$, and $C = -16$. The vertex is at

$$\begin{aligned} \left(\frac{-B}{2A}, f\left(\frac{-B}{2A}\right)\right) &= \left(\frac{-20}{2(-5)}, f\left(\frac{-20}{2(-5)}\right)\right) \\ &= \left(\frac{-20}{-10}, f\left(\frac{-20}{-10}\right)\right) \\ &= (2, f(2)) \\ &= (2, 5(2^2) + 20(2) - 16) \\ &= (2, 5(4) + 40 - 16) \\ &= (2, 20 + 24) \\ &= \boxed{(2, 44)} \end{aligned}$$

A is negative, so this is a $\boxed{\text{maximum}}$.

13. (4 points) Find the equation of a parabola at vertex $(\frac{1}{2}, -3)$ and passing through the point $(\frac{3}{2}, 9)$

Solution: No form is given, so I put this in standard form. I first plug in the vertex to get $y = a(x - \frac{1}{2})^2 - 3$. Then I plug in the point and solve for a:

$$\begin{aligned} 9 &= a(\frac{3}{2} - \frac{1}{2})^2 - 3 \\ 9 &= a(\frac{2}{2})^2 - 3 \\ 9 &= a(1)^2 - 3 \\ 12 &= a \end{aligned}$$

Then I plug a and the vertex in to get $\boxed{y = 12(x - \frac{1}{2})^2 - 3}$.

14. (4 points) Solve the absolute value inequality $|7x - 15| < 3$

Solution: Since $c > 0$, this is equivalent to

$$-3 < 7x - 15 < 3$$

which solves as:

$$12 < 7x < 18$$

$$\boxed{\frac{12}{7} < x < \frac{18}{7}}$$

15. (4 points) Find an absolute value inequality that satisfies the interval $[-12, 18]$.

Solution: This is in the form $|x| \leq c$, where $c > 0$, so I can write this as: $-12 \leq x \leq 18$. I then subtract: $\frac{-12+18}{2} = \frac{6}{2} = 3$, to all sides and get

$$-15 \leq x - 3 \leq 15$$

so my final solution is $\boxed{|x - 3| \leq 15}$.

16. (6 points) Evaluate

$$f(x) = \begin{cases} 5x^2 - 1 & \text{if } x \geq 1 \\ 3 & \text{if } -1 \leq x < 1 \\ 9x & \text{if } x < -1 \end{cases}$$

at $f(3)$, $f(1)$, $f(0)$, $f(-2)$, and $f(-3)$.

Solution: $3 \geq 1$, so

$$\begin{aligned} f(3) &= 5(3^2) - 1 \\ &= 5(9) - 1 \\ &= 45 - 1 \\ &= \boxed{44} \end{aligned}$$

$1 \geq 1$ so

$$\begin{aligned} f(1) &= 5(1^2) - 1 \\ &= 5 - 1 \\ &= \boxed{4} \end{aligned}$$

$-1 \leq 0 < 1$, so

$$f(0) = \boxed{3}$$

$-3 < -1$, so

$$\begin{aligned} f(-3) &= 9(-3) \\ &= \boxed{-27} \end{aligned}$$

$-2 < -1$, so

$$\begin{aligned} f(-2) &= 9(-2) \\ &= \boxed{-18} \end{aligned}$$

17. (6 points) Find the Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

of $f(x) = 6x^2 - 7x + 3$

Solution:

$$\begin{aligned}
&= \frac{6(x+h)^2 - 7(x+h) + 3 - (6x^2 - 7x + 3)}{h} \\
&= \frac{6(x^2 + 2xh + h^2) - 7x - 7h + 3 - 6x^2 + 7x - 3}{h} \\
&= \frac{6x^2 + 12xh + 6h^2 - 7x - 7h + 3 - 6x^2 + 7x - 3}{h} \\
&= \frac{12xh + 6h^2 - 7h}{h} \\
&= \frac{h(12x + 6h - 7)}{h} \\
&= \boxed{12x + 6h - 7}
\end{aligned}$$

18. (6 points) Find the equation of the line in point-slope form, point-intercept form, and standard form that passes through $(-3, 15)$ and $(5, 12)$

Solution: I first find the slope with $x_1 = -3$, $y_1 = 15$, $x_2 = 5$, and $y_2 = 12$

$$m = \frac{15-12}{-3-5} = \frac{3}{-8}$$

Then I can write in point-slope form as $\boxed{y - 15 = \frac{-3}{8}(x+3)}$. I solve for y to write in point-intercept form:

$$\begin{aligned}
y &= \frac{-3}{8}(x+3) + 15 \\
y &= \frac{-3}{8}x - \frac{9}{8} + 15 \\
y &= \frac{-3}{8}x - \frac{9}{8} + \frac{120}{8} \\
&\boxed{y = \frac{-3}{8}x + \frac{111}{8}}
\end{aligned}$$

Lastly, I put this in standard form:

$$\begin{aligned}
y + \frac{3}{8}x + \frac{111}{8} &= 0 \\
8y + 3x + 111 &= 0 \\
&\boxed{3x + 8y - 111 = 0}
\end{aligned}$$

19. (6 points) Find the Average Rate of Change from $x = 1$ to $x = 9$ of

$$f(x) = \frac{3x^2 + 5x}{2}$$

Solution: I choose $a = 1$ and $b = 9$. The Average Rate of Change formula is

$$\begin{aligned}
\frac{\frac{3(1)^2+5(1)}{2} - \left(\frac{3(9^2)+5(9)}{2}\right)}{1-9} &= \frac{\frac{3+5}{2} - \left(\frac{3(81)+45}{2}\right)}{-8} \\
&= \frac{\frac{8}{2} - \left(\frac{243+45}{2}\right)}{-8} \\
&= \frac{4 - \left(\frac{288}{2}\right)}{-8} \\
&= \frac{4 - 144}{-8} \\
&= \frac{-140}{-8} \\
&= \boxed{\frac{35}{2}}
\end{aligned}$$

20. (6 points) Find the complete solution set of $|5x + \frac{1}{3}| = 2$

Solution: I set up two equations:

$$\begin{aligned}
5x + \frac{1}{3} &= 2 \\
5x &= \frac{6}{3} - \frac{1}{3} \\
5x &= \frac{5}{3} \\
\underline{x} &\underline{= \frac{1}{3}}
\end{aligned}
\qquad\qquad
\begin{aligned}
-(5x + \frac{1}{3}) &= 2 \\
-5x &= 2 + \frac{1}{3} \\
-5x &= \frac{6}{3} + \frac{1}{3} \\
-5x &= \frac{7}{3} \\
\underline{x} &\underline{= -\frac{7}{15}}
\end{aligned}$$

I check both solutions in my original equation:

$$\begin{aligned}
|5(\tfrac{1}{3}) + \tfrac{1}{3}| &= 2 \\
|\tfrac{5}{3} + \tfrac{1}{3}| &= 2 \\
|\tfrac{6}{3}| &= 2 \\
|2| &= 2 \\
2 &= 2
\end{aligned}
\qquad\qquad
\begin{aligned}
|5(\tfrac{-7}{15}) + \tfrac{1}{3}| &= 2 \\
|\tfrac{-7}{3} + \tfrac{1}{3}| &= 2 \\
|\tfrac{-6}{3}| &= 2 \\
|-2| &= 2 \\
2 &= 2
\end{aligned}$$

Both solutions work so the answers are $\boxed{\frac{1}{3} \text{ and } -\frac{7}{15}}$.

21. (6 points) Find the complete solution set of $|7x - 2| = -9x$

Solution: I set up two equations:

$$7x - 2 = -9x \qquad -7x + 2 = -9x$$
$$-2 = -16x \qquad 2 = -2x$$
$$\underline{x = \frac{1}{8}} \qquad \underline{-1 = x}$$

I check both solutions:

$$|7(-1) - 2| = -9(-1) \qquad |7(\frac{1}{8}) - 2| = -9(\frac{1}{8})$$
$$|-7 - 2| = 9 \qquad |\frac{7}{8} - 2| = -\frac{9}{8}$$
$$|-9| = 9 \qquad |\frac{7}{8} - \frac{16}{8}| = -\frac{9}{8}$$
$$9 = 9 \qquad |-\frac{9}{8}| = -\frac{9}{8}$$
$$\frac{9}{8} = -\frac{9}{8}$$

Only the solution of $\boxed{x = -1}$ works.

22. (6 points) Find the complete solution set of $|15x + 2| = |2x + 5|$

Solution: I first set up two equations:

$$15x + 2 = 2x + 5 \qquad -(15x + 2) = 2x + 5$$
$$13x + 2 = 5 \qquad -15x - 2 = 2x + 5$$
$$13x = 3 \qquad -15x = 2x + 7$$
$$\underline{x = \frac{3}{13}} \qquad -17x = 7$$
$$\underline{x = -\frac{7}{17}}$$

Now I check both solutions with the original equation:

$$|15(\frac{3}{13}) + 2| = |2(\frac{3}{13}) + 5| \qquad |15(-\frac{7}{17}) + 2| = |2(-\frac{7}{17}) + 5|$$
$$|\frac{45}{13} + \frac{26}{13}| = |\frac{6}{13} + \frac{65}{13}| \qquad |\frac{-105}{17} + \frac{34}{17}| = |\frac{-14}{17} + \frac{85}{17}|$$
$$|\frac{71}{13}| = |\frac{71}{13}| \qquad |\frac{-71}{17}| = |\frac{71}{17}|$$
$$\frac{71}{13} = \frac{71}{13} \qquad \frac{71}{17} = \frac{17}{17}$$

Both solutions work, so the answer is $\boxed{x = \frac{3}{13} \text{ and } x = -\frac{7}{17}}$.