Homework 1 Solutions

1 (a) $4x^2-25$

- Use Difference of Squares

$\boxed{(2x-5)(2x+5)}$

(b) $2x^2+5x-12$

There's a 2 for the coefficient of x^2, so we use the quadratic formula

$$x=\frac{-5\pm\sqrt{25-4(2)(-12)}}{2(2)}$$

$$x=\frac{-5\pm\sqrt{25+96}}{4}$$

$$x=\frac{-5\pm\sqrt{121}}{4}$$

$$x=\frac{-5\pm 11}{4}$$

$$x=\frac{-5-11}{4} \qquad x=\frac{-5+11}{4}$$

$$x=-\frac{16}{4} \qquad x=\frac{6}{4}$$

$$x=-4 \qquad x=\frac{3}{2}$$

$\boxed{(x+4)(2x-3)}$

(c) $x^3-3x^2-4x+12$

- Factor by grouping

$(x^3-3x^2)(-4x+12)$

$x^2(x-3)-4(x-3)$

$(x^2-4)(x-3)$

Difference of squares

$\boxed{(x-2)(x+2)(x-3)}$

(d) x^4+27x

- Reduce to a lower degree

$x(x^3+27)$

Difference of Cubes

$\boxed{x(x+3)(x^2-3x+9)}$

(e) $3x^{3/2}-9x^{1/2}+6x^{-1/2}$

- Reduce to a better degree

$x^{-1/2}(3x^{3/2-(-1/2)}-9x^{1/2-(-1/2)}+6)$

$x^{-1/2}(3x^{4/2}-9x^{2/2}+6)$

$x^{-1/2}(3x^2-9x+6)$

$3x^{-1/2}(x^2-3x+2)$

$\boxed{3x^{-1/2}(x-2)(x-1)}$

(f) x^3y-4xy

$xy(x^2-4)$

$\boxed{xy(x-2)(x+2)}$

2. (a) $\dfrac{x^2+3x+2}{x^2-x-2} = \dfrac{(x+2)(x+1)}{(x+1)(x-2)}$

$= \boxed{\dfrac{x+2}{x-2} \quad \text{if } x \neq -1}$

(b) $\dfrac{2x^2-x-1}{x^2-9} \cdot \dfrac{x+3}{2x+1}$

- First let's factor $2x^2-x-1$:

$x = \dfrac{1 \pm \sqrt{1-4(2)(-1)}}{2 \cdot 2}$

$= \dfrac{1 \pm \sqrt{1+8}}{4}$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1+3}{4} \quad \text{or} \quad x = \frac{1-3}{4}$$

$$x = \frac{4}{4} \qquad x = \frac{-2}{4}$$

$$x = 1 \qquad x = -\frac{1}{2}$$

$$(x-1)(2x+1)$$

Now back to the problem:

$$\frac{2x^2-x-1}{x^2-9} \cdot \frac{x+3}{2x+1} = \frac{(x-1)\cancel{(2x+1)}}{(x-3)\cancel{(x+3)}} \cdot \frac{\cancel{(x+3)}}{\cancel{(2x+1)}}$$

$$= \boxed{\frac{x-1}{x-3} \quad \text{if } x \neq -\tfrac{1}{2} \text{ or } -3}$$

(c) $$\frac{x^2}{x^2-4} - \frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2}$$

Now I make a common denominator

$$= \frac{x^2}{(x-2)(x+2)} - \frac{(x+1)(x-2)}{(x+2)(x-2)}$$

$$= \frac{x^2-(x+1)(x-2)}{(x-2)(x+2)}$$

$$= \frac{x^2-(x^2-x-2)}{(x-2)(x+2)}$$

$$= \frac{x^2-x^2+x+2}{(x-2)(x+2)}$$

$$= \frac{\cancel{x+2}}{(x-2)\cancel{(x+2)}}$$

$$= \boxed{\frac{1}{x-2} \quad \text{if } x \neq -2}$$

(a) $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{1}{y} - \dfrac{1}{x}} = \dfrac{\dfrac{x^2}{xy} - \dfrac{y^2}{xy}}{\dfrac{x}{xy} - \dfrac{y}{xy}}$

$$= \frac{\dfrac{x^2-y^2}{xy}}{\dfrac{x-y}{xy}}$$

$$= \frac{x^2-y^2}{xy} \div \frac{x-y}{xy}$$

$$= \frac{x^2-y^2}{\cancel{xy}} \cdot \frac{\cancel{xy}}{x-y}$$

$$= \frac{x^2-y^2}{x-y}$$

$$= \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}}$$

$$= \boxed{x+y \quad \text{if } x \neq y}$$

3. (a) $(p+q)^2 = p^2+q^2$?

$\boxed{\text{FALSE.}}$

$(p+q)^2 = (p+q)(p+q)$

$= p^2 + 2pq + q^2$

(b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

I accepted Both answers here

I list it as a property of radicals, but be careful!

It is not true if you consider:

$\sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1} \cdot \sqrt{-1}$ DNE

(c) $\boxed{\text{False}}$ $\sqrt{a^2+b^2} \neq a+b$

Let $a=1$ $b=1$. Then
$\sqrt{1^2+1^2} = \sqrt{2} \neq 1+1=2$

(d) $\frac{1+TC}{C} = 1+T$?

$\boxed{\text{FALSE}}$ $\frac{1+TC}{C} = C(1+T)$

$1+TC = C+TC$ X

(e) $\frac{1}{x-y} = \frac{1}{x} - \frac{1}{y}$? $\boxed{\text{FALSE}}$

Let $x=2$ $y=1$

$\frac{1}{2-1} = \frac{1}{1} = 1 \neq \frac{1}{2} - \frac{1}{1} = -\frac{1}{2}$

(f) $\dfrac{\frac{1}{x}}{\frac{a}{x} - \frac{b}{x}} = \dfrac{\frac{1}{x}}{\frac{a-b}{x}} = \frac{1}{x} \cdot \frac{x}{a-b}$

$= \frac{1}{a-b}$ $\boxed{\text{TRUE}}$

4. $\boxed{\text{No!}}$ $f(1) = \frac{1^2-1}{1-1} = \frac{0}{0} =$ DNE

$g(1) = 1$

The domain is different

5.(a) $\boxed{f(-4) = -2 \quad g(3) = 4}$

(b) $f(x) = g(x)$ at $\boxed{x = -2 \ \& \ x = 2}$

(c) $f(x) = -1$ at $\boxed{x = -3 \ \& \ x = 4}$

(d) f is decreasing on $\boxed{[0, 4]}$

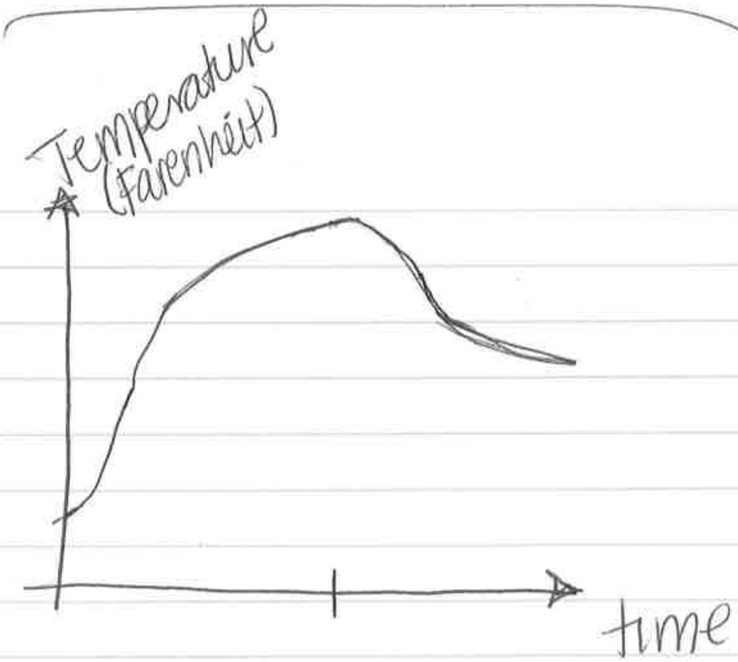
(e) Domain: [-4, 4]
Range: [-2, 3]

(f) Domain: [-4, 3]
Range: [.5, 4]

6. Runner A at first was the slowest, with Runner B in the lead. Around 7 seconds, A speed up, passes Runner B (whose fallen to 2nd) and passes Runner C in the lead. About 5 more seconds after, the race was over. A won. A few seconds after B passes C before the finish line. C comes in 3rd.

A won | Yes, each runner completed the race

7. First, I take the pie out of the oven. It is frozen so it immediately starts to thaw & rise in temperature. Then it put it in the oven & it rises in temperature in the oven for a hour. Then I take it out of the oven, & the temperature begins to drop again but never reaching the cold temperature it was before.



8. $f(x) = \dfrac{2x^3-5}{x^2+x-6}$

$x^2+x-6 = (x+3)(x-2)$

$x=-3$ & $x=2$

Domain: $x \neq -3$ & $x \neq 2$

$\boxed{(-\infty,-3) \cup (-3,2) \cup (2,\infty)}$

or

$\boxed{x<-3 \ \& \ -3<x<2 \ \& \ x>2}$

9. $g(t) = \sqrt{3-t} - \sqrt{2+t}$

$3-t \geq 0$ $\qquad$ $2+t \geq 0$

$3 \geq t$ $\qquad$ $t \geq -2$

$-2 \leq t \leq 3$

OR

$[-2,3]$