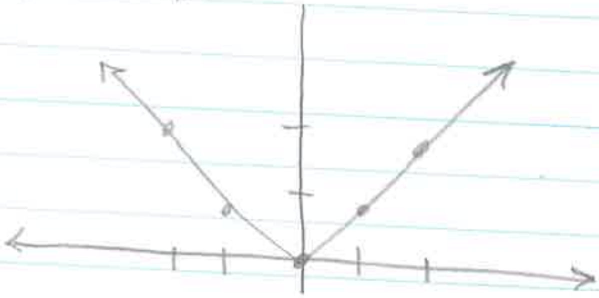
① p. 211 #24

$f(x) = |x|$



absolute max: $\boxed{\text{none}}$

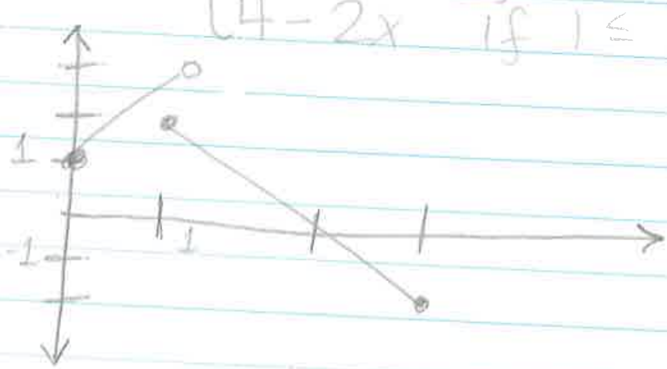
absolute min: $\boxed{f(0)=0}$

local max: $\boxed{\text{none}}$

local min: $\boxed{f(0)=0}$

② p. 211 #28

$$f(x) = \begin{cases} 2x+1 & \text{if } 0 \le x < 1 \\ 4-2x & \text{if } 1 \le x \le 3 \end{cases}$$



absolute max: $\boxed{\text{none}}$

absolute min: $\boxed{f(3)=-2}$

local max: $\boxed{\text{none}}$

local min: $\boxed{\text{none}}$

(3) p. 211 #30

$f(x) = x^3 + 6x^2 - 15x$

$f'(x) = 3x^2 + 12x - 15$

$3x^2 + 12x - 15 = 0$

$3(x^2 + 4x - 5) = 0$

$3(x+5)(x-1) = 0$

$\boxed{x = -5 \quad x = 1}$

(4) p. 211 #34

$g(t) = |3t-4| = \begin{cases} 3t-4 & \text{if } 3t-4 \geq 0 \\ -(3t-4) & \text{if } 3t-4 < 0 \end{cases}$

$= \begin{cases} 3t-4 & \text{if } 3t \geq 4 \\ -3t+4 & \text{if } 3t < 4 \end{cases}$

$g(t) = \begin{cases} 3t-4 & \text{if } t \geq 4/3 \\ -3t+4 & \text{if } t < 4/3 \end{cases}$

$g'(t) = \begin{cases} 3 & \text{if } t > 4/3 \\ -3 & \text{if } t < 4/3 \end{cases}$

$g'(t) = 0$ nowhere!

$g'(t)$ DNE at $\boxed{t = 4/3}$ (we have a corner V)

(5) p. 212 #38

$g(x) = \sqrt[3]{4-x^2} = (4-x^2)^{1/3}$

$g'(x) = \frac{1}{3}(4-x^2)^{-2/3} \cdot (-2x)$

$= \dfrac{-2x}{3(4-x^2)^{2/3}}$

$g'(x) = 0 \quad \dfrac{-2x}{3(4-x^2)^{2/3}} = 0$

$-2x = 0$

$\boxed{x = 0}$

$g'(x)$ DNE

p. 2

$$3(4-x^2)^{2/3}=0$$
$$(4-x^2)^{2/3}=0$$
$$4-x^2=0$$
$$4=x^2$$
$$\boxed{x=\pm 2}$$

6) p. 212 # 46

$f(x)=5+54x-2x^3,\ [0,4]$

1) $f'(x)=54-8x^2$

$$54-8x^2=0$$
$$54=8x^2$$
$$\frac{54}{8}=x^2$$
$$x=\pm\sqrt{\frac{54}{8}}=\pm\sqrt{\frac{27}{4}}=\pm\frac{\sqrt{27}}{2}=\pm\frac{3\sqrt{3}}{2}$$

$$f\left(\frac{3\sqrt{3}}{2}\right)=5+54\left(\frac{3\sqrt{3}}{2}\right)-2\left(\frac{3\sqrt{3}}{2}\right)^3$$
$$=5+27\cdot 3\sqrt{3}-\frac{2\cdot 27\cdot 3\sqrt{3}}{8}$$
$$=5+81\sqrt{3}-\frac{81}{4}\sqrt{3}$$
$$=5+\frac{243}{4}\sqrt{3}\approx 110$$

$$\begin{array}{r} 27 \\ \times 3 \\ \hline 81 \\ \times 3 \\ \hline 243 \end{array}$$

$$f\left(\frac{-3\sqrt{3}}{2}\right)=5+54\left(\frac{-3\sqrt{3}}{2}\right)-2\left(\frac{-3\sqrt{3}}{2}\right)^3$$
$$=5-81\sqrt{3}-\frac{2(-27)3\sqrt{3}}{8}$$
$$=5-81\sqrt{3}+\frac{81}{4}\sqrt{3}$$
$$=5-\frac{243}{4}\sqrt{3}\approx -100$$

(2) $f(0) = 5 + 54 \cdot 0 - 2 \cdot 0^3$

$f(0) = 5$

$f(4) = 5 + 54 \cdot 4 - 2 \cdot 4^3$

$= 5 + 216 - 128$

$f(4) = 93$

216
5
221
128
93

(3) absolute max: $\boxed{f(\sqrt{3})}$

absolute min: $\boxed{f(-\sqrt{3})}$

(7) p. 212 # 54

$f(t) = \frac{\sqrt{t}}{1+t^2}$ $[0, 2]$

$f(t) = \frac{t^{1/2}}{1+t^2}$

$f'(t) = \frac{\frac{1}{2}t^{-1/2}(1+t^2)(2t)t^{1/2}}{(1+t^2)}$

$= \frac{\frac{1}{2}t^{-1/2} + \frac{1}{2}t^2 t^{3/2}}{(1+t^2)}$

$= \frac{\frac{1}{2}t^{-1/2} - \frac{3}{2}}{(1+t^2)^2}$

$f'(t) = 0$ $\frac{1}{2}t^{-1/2} = 0$

$\frac{1}{2}t^{-1/2}\ t^{3/2}$

$t^{-1/2}\ {}^{3/2}$

$\frac{t^{-1/2}}{t^{3/2}}$

t^{-2}

$\frac{1}{t^2}$

$1 =$

$\frac{1}{3}$

$t = \pm$

p. 4

$f'(t)$ DNE $\quad (1+t^2)^2 = 0$

$1+t^2 = 0$

$t^2 = -1 \quad X$

$f(\frac{1}{3}) = \dfrac{\sqrt{\frac{1}{3}}}{1+(\sqrt{\frac{1}{3}})^2} = \dfrac{1}{\sqrt{3}(1+\frac{1}{3})} = \dfrac{1}{\sqrt{3}(\frac{4}{3})}$

$= \dfrac{3}{4\sqrt{3}} \approx 0.433$

$f(-\frac{1}{3}) = \dfrac{\sqrt{-\frac{1}{3}}}{1+(\sqrt{-\frac{1}{3}})^2}$ DNE

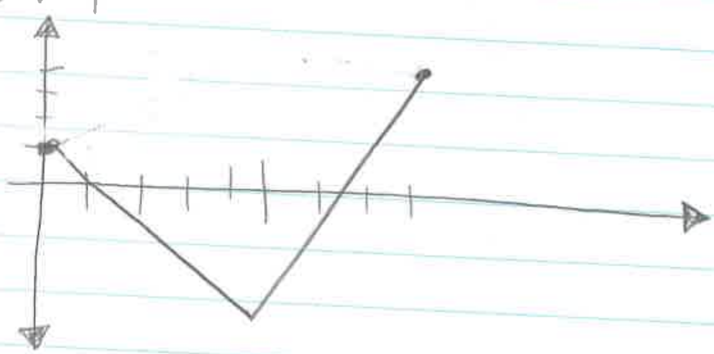
2 $\quad f(0) = \dfrac{\sqrt{0}}{1+0^2} = \dfrac{0}{1} = 0$

$f(2) = \dfrac{\sqrt{2}}{1+(\sqrt{2})^2} = \dfrac{\sqrt{2}}{1+2} = \dfrac{\sqrt{2}}{3} \approx 0.47$

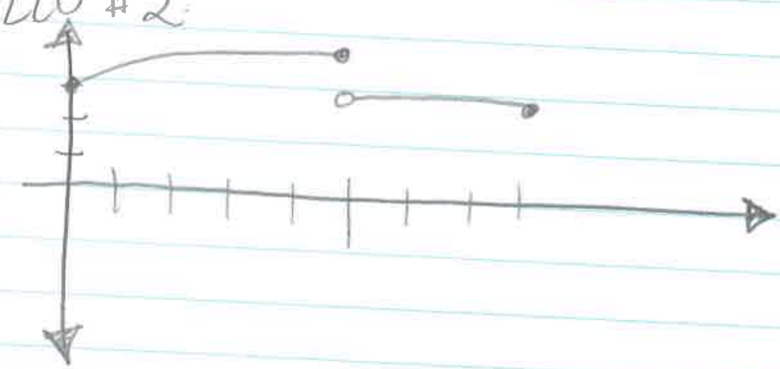
3 absolute max: $f(2)$
absolute min: $f(0)$

8 p. 220 #4

switch these



9 p. 220 #2



(10) p. 220 #6

$f(x) = x^3 - 2x^2 - 4x + 2 \quad [-2, 2]$

(1) f is a polynomial, so its continuous on $[-2, 2]$

(2) f is a polynomial, so its defferentiable on $[-2, 2]$

(3) $f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 2$

$= -8 - 2 \cdot 4 + 8 + 2$

$= -8 - 8 + 8 + 2$

$f(-2) = -6$

$f(2) = 2^3 - 2 \cdot 2^2 - 4 \cdot 2 + 2$

$= 8 - 8 - 8 + 2$

$f(2) = -6$

$f'(x) = 3x^2 - 4x - 4$

$f'(c) = 3c^2 - 4 \cdot c - 4 = 0$

$c = \dfrac{4 \pm \sqrt{16 - 4(3)(-4)}}{2 \cdot 3}$

$c = \dfrac{4 \pm \sqrt{16 + 48}}{6}$

$c = \dfrac{4 \pm \sqrt{64}}{6}$

$c = \dfrac{4 \pm 8}{6}$

$c = \dfrac{4 + 8}{6} \qquad c = \dfrac{4 - 8}{6}$

$c = \dfrac{12}{6} \qquad c = -\dfrac{4}{6}$

$\boxed{c = 2 \qquad c = -\dfrac{2}{3}}$

$\begin{array}{r} 16 \\ \times 3 \\ \hline 48 \\ +16 \\ \hline 64 \end{array}$

p. 6

(11) p. 220 # 10

$f(x) = \tan(x)$

$f(0) = \tan(0) = 0$

$f(\pi) = \tan(\pi) = 0$

$f'(x) = \sec^2(x) = \dfrac{1}{\cos^2(x)}$

$f'(c) = \sec^2(c) = \dfrac{1}{\cos^2(c)} = 0$

$1 = 0$ x

No such c. However, f is not continuous on $(0, \pi)$ so it doesn't contradict Rolle's Theorem

(12) p. 220 # 20

$2x - 1 - \sin(x) = 0$

$f(x) = 2x - 1 - \sin(x)$

$f(0) = 2 \cdot 0 - 1 - \sin(0) = -1 < 0$

$f(2\pi) = 2(2\pi) - 1 - \sin(2\pi) = 4\pi - 1 > 0$

So by the Intermediate Value Theorem, there is at least one number c since $f(c) = 0$. Now I show there's only one.

Assume there are at least two, a & b where $f(a) = f(b) = 0$. f is continuous & differentiable, so by Rolle's Theorem, there exists a c such that $f'(c) = 0$. Let's find it:

$f'(x) = 2 - \cos(x)$

$f'(c) = 2 - \cos(c) = 0$

$-\cos(c) = -2$

$\cos(c) = 2$

However, $-1 \leq \cos(x) \leq 1$, so there is no such c. So there is only one root.

(13) p. 220 # 26

Since f' exists, f is continuous & differentiable. So by the Mean Value Theorem,

$f'(c)(b-a) = f(b) - f(a)$

So let $b=8$ & $a=2$.

$f'(c)(8-2) = f(8) - f(2)$

$6f'(c) = f(8) - f(2)$

$3 \le f'(x) \le 5$

$18 \le 6 \cdot f'(c) \le 30$

$18 \le f(8) - f(2) \le 30$

(14) p. 228 # 8

(a) $[0,4) \cup (6,8)$ this is where $f' > 0$

(b) $x=4$ f' changes from >0 to <0 so

$\boxed{x=4 \text{ is a local max}}$

$x=6$ f' changes from <0 to >0 so

$\boxed{x=6 \text{ is a local min}}$

(c) $\boxed{\begin{array}{l}\text{Concave up: } [0,1) \cup (2,3) \cup (5,7) \\ \text{Concave down: } (1,2) \cup (3,5) \cup (7,\infty)\end{array}}$

CU when slope of tangent lines of f' is positive

CD when slope of tangent lines of f' is negative

(d) inflection points: $\boxed{x=1,\ x=2,\ x=3,\ x=5}$

where concavity changes

(15) p. 228 #10

(a) $f(x) = 2x^3 - 9x^2 + 12x - 3$

$f'(x) = 6x^2 - 18x + 12$

$= 6(x^2 - 3x + 2)$

$= 6(x-2)(x-1)$

p. 8

	Interval	$6(x-2)$	$(x-1)$	$=f'(x)$	I/D
0:	$(-\infty,1)$	$-$	$-$	$+$	I
3/2:	$(1,2)$	$-$	$+$	$-$	D
3:	$(2,\infty)$	$+$	$+$	$+$	I

$\boxed{\text{Increasing: } (-\infty,1)\cup(2,\infty)}$

$\boxed{\text{Decreasing: } (1,2)}$

(b) $\boxed{\text{local max: } x=1}$

$\boxed{\text{local min: } x=2}$

(c) $f''(x) = 12x-18$

$12x-18=0$

$6(2x-3)=0$

	Intervals	$6(2x-3)$	$=f''(x)$	Concavity
0:	$(-\infty,3/2)$	$-$	$-$	CD
2:	$(3/2,\infty)$	$+$	$+$	CU

$\boxed{\text{CU: } (3/2,\infty)}$

$\boxed{\text{CD: } (-\infty,3/2)}$

inflection point: $\boxed{x=3/2}$

(16) p. 228 #12

$f(x)=\dfrac{x}{x^2+1}$

$f'(x)=\dfrac{1(x^2+1)-x(2x)}{(x^2+1)^2}$

$f'(x)=\dfrac{x^2+1-2x^2}{(x^2+1)^2}=\dfrac{-x^2+1}{(x^2+1)^2}$

$\dfrac{-x^2+1}{(x^2+1)^2}=0 \qquad -x^2+1=0 \qquad -x^2=-1$

$x^2 = 1$, $x = \pm 1$

Intervals	$\frac{-x^2+1}{(x^2+1)^2}$	$= f'(x)$	I/D
-2: $(-\infty, -1)$	$\frac{-}{+}$	$-$	D
0: $(-1, 1)$	$\frac{+}{+}$	$+$	I
2: $(1, \infty)$	$\frac{-}{+}$	$-$	D

$\boxed{\text{increasing: } (-1,1) \quad \text{decreasing: } (-\infty,-1) \cup (1,\infty)}$

(b) $\boxed{\text{local max: } x = 1}$

$\boxed{\text{local min: } x = -1}$

(c) $$f''(x) = \frac{-2x(x^2+1)^2 - (-x^2+1)(2(x^2+1)2x)}{(x^2+1)^4}$$

$$= \frac{-2x(x^4+2x^2+1) - (-x^2+1)(2x^2+2)(2x)}{(x^2+1)^4}$$

$$= \frac{-2x^5 - 4x^3 - 2x - (-2x^3+2x)(2x^2+2)}{(x^2+1)^4}$$

$$= \frac{-2x^5 - 4x^3 - 2x - (-4x^5 - 4x^3 + 4x^3 + 4x)}{(x^2+1)^4}$$

$$= \frac{-2x^5 - 4x^3 - 2x + 4x^5 + \cancel{4x^3} - \cancel{4x^3} - 4x}{(x^2+1)^4}$$

$$= \frac{2x^5 - 4x^3 - 6x}{(x^2+1)^4}$$

$$= \frac{2x(x^4 - 2x^2 - 3)}{(x^2+1)^4}$$

$$= \frac{2x(x^2-3)(x^2+1)}{(x^2+1)^4}$$

$$= \frac{2x(x^2-3)}{(x^2+1)^3}$$

$$= \frac{2x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3} = 2x(x-\sqrt{3})(x+\sqrt{3})\left(\frac{1}{(x^2+1)^3}\right)$$

Intervals	$2x$	$(x-\sqrt{3})$	$(x+\sqrt{3})$	$1/(x^2+1)^3$	$=f''(x)$	Concavity
$(-\infty,-\sqrt{3})$	−	−	−	+	−	CD
$(-\sqrt{3},0)$	−	−	+	+	+	CU
$(0,\sqrt{3})$	+	−	+	+	−	CD
$(\sqrt{3},\infty)$	+	+	+	+	+	CU

CU: $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$
CD: $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$

Inflection points: $x=-\sqrt{3}$, $x=0$, $x=\sqrt{3}$

17 p. 228 #14

$f(x)=\cos^2(x)-2\sin(x) \quad 0\le x\le 2\pi$

(a) $f'(x)=-2\cos(x)\sin(x)-2\cos(x)$

$-2\cos(x)\sin(x)-2\cos(x)=0$

$-2\cos(x)\sin(x)=2\cos(x)$

$\sin(x)=-1$

$x=3\pi/2$

	Intervals	$\cos^2(x)-2\sin(x)$	$=f'(x)$	I/D
π	$[0,3\pi/2)$	+	+	I
$7\pi/4$	$(3\pi/2,2\pi]$	−	−	D

increasing: $[0,3\pi/2)$
decreasing: $(3\pi/2,2\pi]$

(b) local max: $x=3\pi/2$

(c) $f''(x)=-2(-\sin(x))\sin(x)+(-2\cos(x))\cos(x)-2(-\sin(x))$

$=2\sin^2(x)-2\cos^2(x)+2\sin(x)$

$2\sin^2(x)-2\cos^2(x)+2\sin(x)=0$

$\sin^2(x)-\cos^2(x)+\sin(x)=0$

Skip This

(18) p. 228 #16

$f(x) = \frac{x^2}{x-1}$

$f'(x) = \frac{2x(x-1) - x^2(1)}{1}$

$= 2x^2 - 2x - x^2$

$f'(x) = x^2 - 2x = x(x-2)$

$x=0 \quad x=2$

Intervals	x	(x-2)	=f'(x)	I/D
$(-\infty, 0)$	−	−	+	I
$(0, 2)$	+	−	−	D
$(2, \infty)$	+	+	+	I

local max: $x=0$
local min: $x=2$

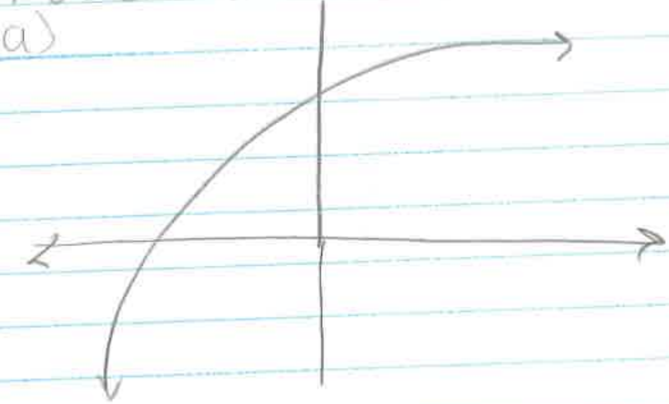
$f''(x) = 2x-2$
$f''(0) = -2$
$f''(2) = 4-2 = 2$

$x=0$ local max
$x=2$ local min

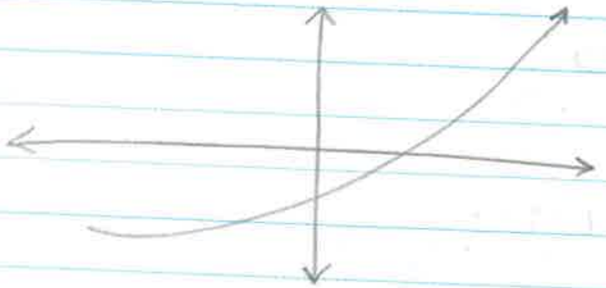
I like the 1st derivative test more because it's definitive.

(19) p. 228 #20

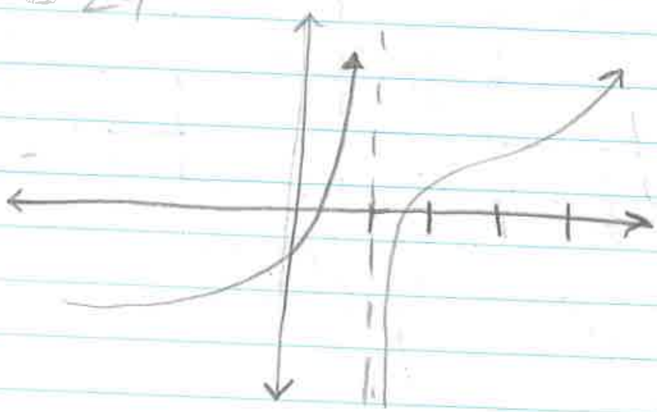
(a)



(b)

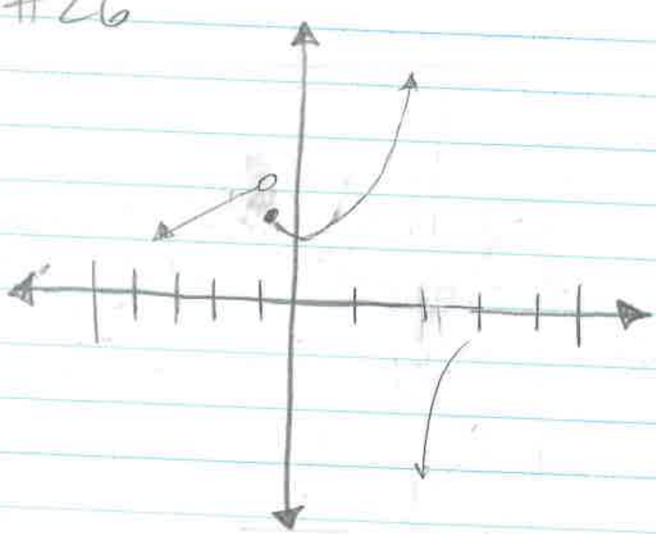


20 P.228 #24



21 P.228 #26

skipped for now



22 P.229 #30

(a) B

(b) E

(c) A

23

p. 229 #38

(a) $h(x) = 5x^3 - 3x^5$

$h'(x) = 15x^2 - 15x^4$

$= 15x^2(1 - x^2)$

$= 15x^2(1-x)(1+x)$

	Intervals	$15x^2$	$(1-x)$	$(1+x)$	$= f'(x)$	I/D
-2	$(-\infty, -1)$	+	+	−	−	D
$-1/2$	$(-1, 0)$	+	+	+	+	I
$1/2$	$(0, 1)$	+	+	+	+	I
2	$(1, \infty)$	+	−	+	−	D

increasing: $(-1,0) \cup (0,1)$
decreasing: $(-\infty,-1) \cup (1,\infty)$

(b) local min: $x = -1$
local max: $x = 1$

(c) $f''(x) = 30x - 60x^3$

$= 30x(1 - 20x^2)$

$= 30x(1 - \sqrt{20}\,x)(1 + \sqrt{20}\,x)$

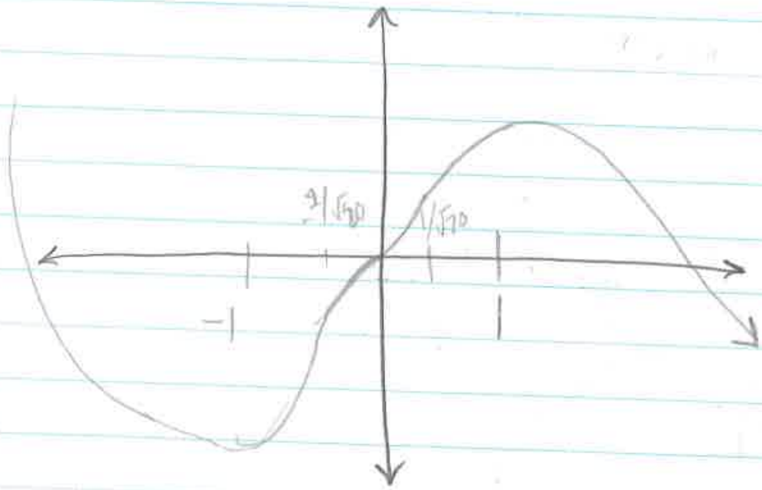
	Intervals	$3x$	$(1-\sqrt{20}\,x)$	$(1+\sqrt{20}\,x)$	$= f''$	Concavity
-2	$(-\infty, -1/\sqrt{20})$	−	+	−	+	CU
$-1/100$	$(-1/\sqrt{20}, 0)$	−	+	+	−	CD
$1/100$	$(0, 1/\sqrt{20})$	+	+	+	+	CU
2	$(1/\sqrt{20}, \infty)$	+	−	+	−	CD

CU: $(-\infty, -1/\sqrt{20}) \cup (0, 1/\sqrt{20})$

CD: $(-1/\sqrt{20}, 0) \cup (1/\sqrt{20}, \infty)$

Inflection points: $\boxed{x = \pm 1/\sqrt{20}, 0}$

(d)



(24) p. 229 # 40

$$G(x) = 5x^{2/3} - 2x^{5/3}$$

$$G'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3}$$

$$\frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3} = 0$$

$$\frac{10}{3}x^{-1/3} = \frac{10}{3}x^{2/3}$$

$$x^{-1/3} = x^{2/3}$$

$1 = x \quad x = 0$

	Intervals	$\frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3}$	$= G'(x)$	I/D
-1:	$(-\infty, 0)$	−	−	D
1/2:	$(0, 1)$	+	+	I
2:	$(1, \infty)$	−	−	D

increasing: $(0, 1)$
decreasing: $(-\infty, 0) \cup (1, \infty)$

(b) local min: $x = 0$
local max: $x = 1$

(c) $G'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3}$

$G''(x) = -\frac{10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3}$

$\frac{10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3} = 0$

$\frac{10}{9}x^{-4/3} = \frac{20}{9}x^{-1/3}$

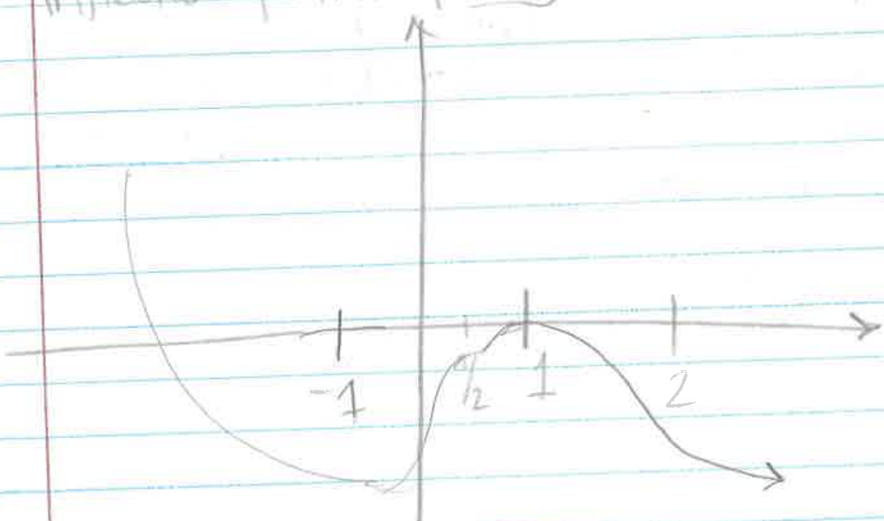
$\frac{1}{2}x^{1/3} = x^{4/3}$

$\frac{1}{2} = x \quad x = 0$

Intervals	$\frac{10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3}$	$= G''(x)$	Concavity
$(-\infty, 0)$	+	+	CU
$(0, 1/2)$	−	−	CD
$(1/2, \infty)$	−	−	CD

$\boxed{\begin{array}{l}CU: (-\infty, 0) \\ CD: (0, 1/2) \cup (1/2, \infty)\end{array}}$

$\boxed{\text{Inflection point: } \boxed{x = 0}}$



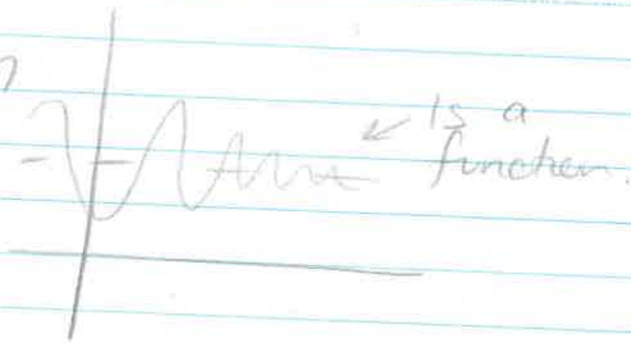
p. 16

25 p.229 # 54

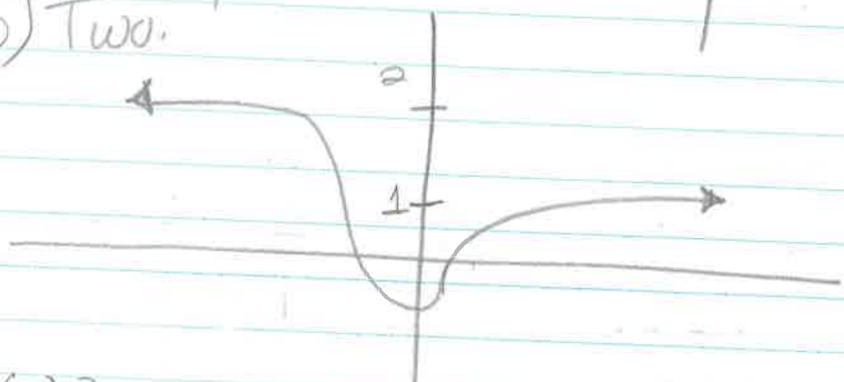
$f' < 0$ "declining"
$f'' < 0$ "slower rate"

26 p.241 # 2

(a) No. Yes.



(b) Two.



27 (a) 2 (b) -1 (c) $-\infty$ (d) $-\infty$ (e) $+\infty$ (f) $y=-1$, $y=2$; $x=0$, $x=2$

28 p.241 # 8

$$\lim_{x\to\infty} \sqrt{\frac{9x^3+8x-4}{3-5x+x^3}} = \lim_{x\to\infty} \sqrt{\frac{9+\frac{8}{x^2}-4/x^3}{3/x^3-5/x^2+1}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{9+8/x^2-4/x^3}}{\sqrt{3/x^3-5/x^2+1}} = \frac{\lim_{x\to\infty}\sqrt{9+8/x^2-4/x^3}}{\lim_{x\to\infty}\sqrt{3/x^3-5/x^2+1}}$$

$$= \frac{\sqrt{9}}{\sqrt{1}} = 3$$

29) p. 241 # 10

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to\infty} \frac{\frac{1}{x^3}-\frac{1}{x}}{1-\frac{1}{x^2}+\frac{1}{x^3}}$$

$$= \boxed{0}$$

30)

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}} = \lim_{x\to\infty} \frac{x^2 \cdot 1/x^{16}}{\sqrt{x^4+1} \cdot 1/x^{16}}$$

$$= \lim_{x\to\infty} \frac{1/x^{14}}{\sqrt{1+1/x^4}}$$

$$= \boxed{0}$$

31)

$$\lim_{x\to\infty} \sqrt{x}\,\sin(1/x) = \boxed{0}$$

32) p. 243 # 40

$$\lim_{x\to\infty} \frac{1+2x^2}{1+x^2} = \lim_{x\to\infty} \frac{\frac{1}{x^2}+2}{1/x^2+1} = 2$$

$$\lim_{x\to-\infty} \frac{1+2x^2}{1+x^2} = \lim_{x\to\infty} \frac{1/x^2+2}{1/x^2+1} = 2$$

$$f'(x) = \frac{4x(1+x^2)-(1+2x^2)(2x)}{(1+x^2)^2}$$

$$= \frac{4x+\cancel{4x^3}-2x-\cancel{4x^3}}{(1+x^2)^2}$$

$$= \frac{2x}{(1+x^2)^2}$$

	Intervals	$f'(x)$	I/D
-1	$(-\infty, 0)$	$-$	D
1	$(0, \infty)$	$+$	I

$$f''(x) = \frac{2(1+x^2)^2 - 2x\,2(1+x^2)(2x)}{(1+x^2)^4}$$

p. 18

$$f''(x) = \frac{2(1+2x^2+x^4) - 8x^2(1+x^2)}{(1+x^2)^4}$$

$$= \frac{2+4x^2+2x^4-8x^2-8x^4}{(1+x^2)^4}$$

$$= \frac{-6x^4-4x^2+2}{(1+x^2)^4}$$

$$= \frac{-2(3x^4+2x^2-1)}{(1+x^2)^4}$$

$$x^2 = \frac{-2 \pm \sqrt{4-4(3)(-1)}}{2 \cdot 3}$$

$$x^2 = \frac{-2 \pm \sqrt{4+12}}{6}$$

$$x^2 = \frac{-2 \pm \sqrt{16}}{6}$$

$$x^2 = \frac{-2 \pm 8}{6}$$

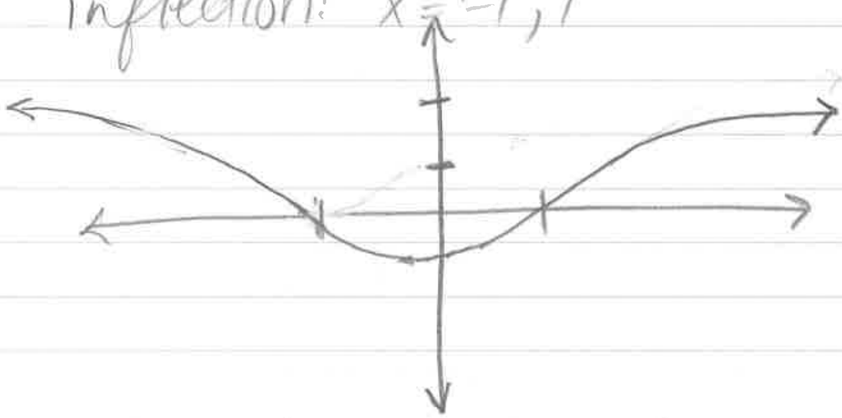
$x^2 = \frac{-10}{6}$ or $x^2 = \frac{6}{6}$

$\cancel{x^2 = -\frac{5}{3}}$ or $x^2 = 1$

$x = \pm 1$ $\quad f''(x) = \frac{-2(3x^2+5)(x^2-1)}{(1+x^2)^4}$

Intervals	$-2(3x^2+5)$	(x^2-1)	$\left(\frac{1}{1+x^2}\right)^4$	$= f''(x)$	Concavity
$(-\infty, -1)$	$-$	$+$	$+$	$-$	CD
$(-1, 1)$	$-$	$-$	$+$	$+$	CU
$(1, \infty)$	$-$	$+$	$+$	$-$	CD

inflection: $x = -1, 1$



(33) p.243 #54

$y = x^3(x+2)^2(x-1)$

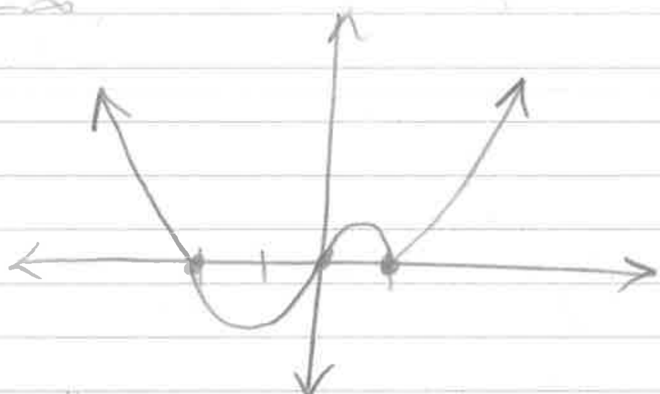
y-intercepts: $0^3(2)^2(-1) = 0$

x-intercepts: $0 = x^3(x+2)^2(x-1)$

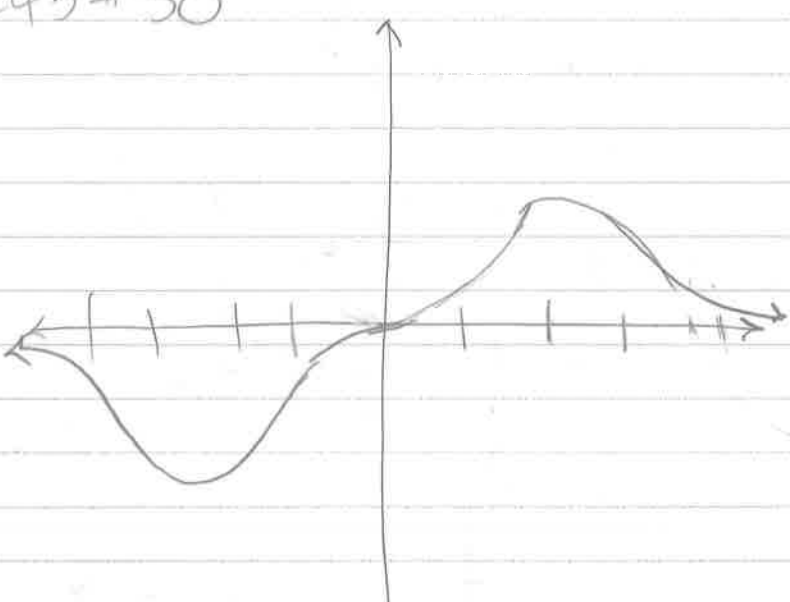
$x=0, x=-2, x=1$

$\lim_{x \to \infty} \; +\infty \; +\infty \; +\infty = \infty$

$\lim_{x \to -\infty} \; -\infty \; +\infty \; -\infty = \infty$



(34) p.243 #58



(35) p.243 #61

p.20

(a) $-1 \le \sin(x) \le 1$

$$\frac{-1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$$

$$\lim_{x\to\infty} \frac{-1}{x} \le \lim_{x\to\infty} \frac{\sin(x)}{x} \le \lim_{x\to\infty} \frac{1}{x}$$

$$0 \le \lim_{x\to\infty} \frac{\sin(x)}{x} \le 0$$

$$\boxed{\lim_{x\to\infty} \frac{\sin(x)}{x} = 0}$$

(b) — skipping. Do not need for final.

36 p. 250 #8

$y = (4-x^2)^5$

A) domain: $(-\infty, \infty)$

B) $0 = (4-x^2)^5$ $\quad$ $y = (4-0^2)^5$

$0 = 4 - x^2$ $\quad$ $y = 4^5$

$x^2 = 4 \rightarrow x = \pm 2$ $\quad$ $y = 1024$

C) $f(-x) = (4-(-x)^2)^5$

$= (4-x^2)^5$ even

D) $\lim_{x\to\infty} (4-x^2)^5 = -\infty$

$\lim_{x\to-\infty} (4-x^2)^5 = -\infty$

No vertical asymptotes

E) $f'(x) = 5(4-x^2)^4(-2x) = -10x(4-x^2)^4$

$x=0$ $\quad x=+2 \quad x=-2$

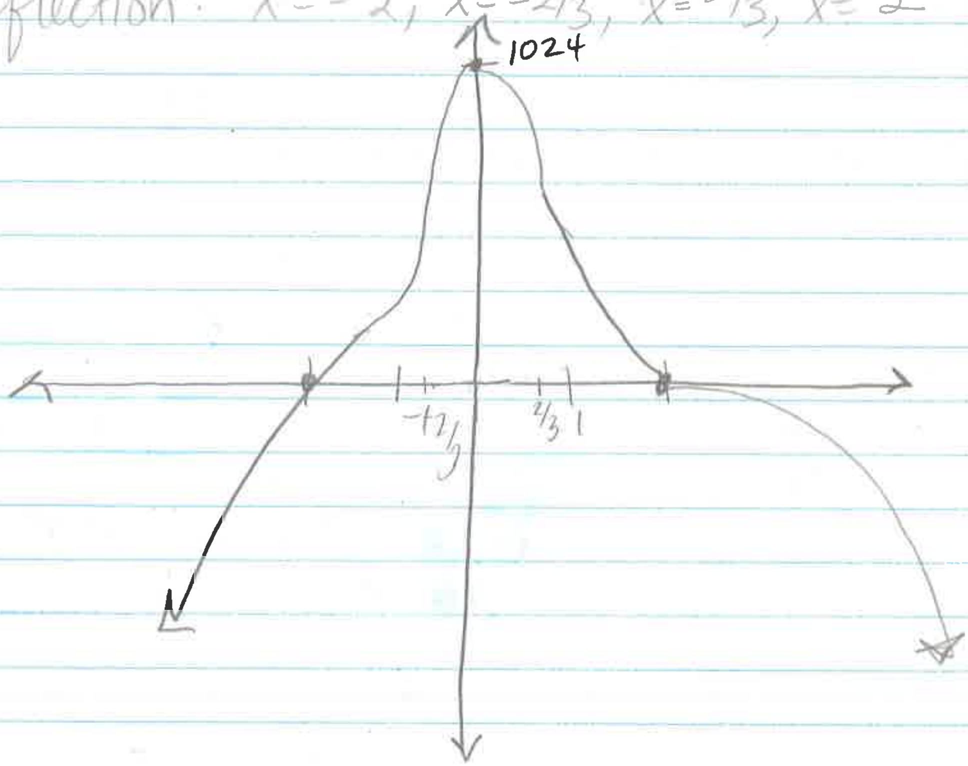
Intervals	$-10x$	$(4-x^2)^4$	$= f'(x)$	I/D
$(-\infty,-2)$	+	+	+	I
$(-2,0)$	+	+	+	I
$(0,2)$	−	+	−	D
$(2,\infty)$	−	+	−	D

p21

(F) Local max: $x = 0$
Local min: none
(G) $f'(x) = -10x(4-x^2)^4$
$f''(x) = -10(4-x^2)^4 + (-10x)4(4-x^2)^3 \cdot (-2x)$
$= -10(4-x^2)^4 + 80x^2(4-x^2)^3$
$= -10(x^2-4)^3(9x^2-4)$
$x = \pm 2$ $x = \pm 2/3$

Intervals	$-10(x^2-4)^3$	$(9x^2-4)$	$= f''(x)$	Concavity
$(-\infty, -2)$	$-$	$+$	$-$	CD
$(-2, -2/3)$	$+$	$+$	$+$	CU
$(-2/3, 2/3)$	$+$	$-$	$-$	CD
$(2/3, 2)$	$+$	$+$	$+$	CU
$(2, \infty)$	$-$	$+$	$-$	CD

Inflection: $x = -2$, $x = -2/3$, $x = 2/3$, $x = 2$



P22

(35) p.250 #14

$$y = \frac{1}{x^2-4}$$

(a) $x^2-4=0 \quad x \neq \pm 2 \quad (-\infty,-2)\cup(-2,2)\cup(2,\infty)$

(b) $0 = \frac{1}{x^2-4} \quad 0 \neq 1$ NO x-intercept

$y = \frac{1}{0^2-4} = \frac{-1}{4} \quad y = -1/4$ y-intercept

(c) $f(-x) = \frac{1}{(-x)^2-4} = \frac{1}{x^2-4} = f(x)$ even

(d) $\lim_{x\to\infty} \frac{1}{x^2-4} = \lim_{x\to\infty} \frac{1/x^2}{1-4/x^2} = 0$

$\lim_{x\to-\infty} \frac{1}{x^2-4} = \lim_{x\to-\infty} \frac{1/x^2}{1-4/x^2} = 0$

$\lim_{x\to 2^-} \frac{1}{x^2-4} = \frac{1}{-0} = -\infty \qquad \lim_{x\to -2^-} \frac{1}{x^2-4} = \frac{1}{+0} = +\infty$

$\lim_{x\to 2^+} \frac{1}{x^2-4} = \frac{1}{+0} = \infty \qquad \lim_{x\to -2^+} \frac{1}{x^2-4} = \frac{1}{-0} = -\infty$

(e) $f(x) = 1/(x^2-4) = (x^2-4)^{-1}$

$f'(x) = -(x^2-4)^{-2}(2x) = \frac{-2x}{(x^2-4)^2} = \frac{-2x}{(x-2)^2(x+2)^2}$

Intervals	$-2x$	$(x-2)^2$	$(x+2)^2$	$f'(x)$	I/D
$(-\infty,-2)$	+	+	+	+	I
$(-2,0)$	+	+	+	+	I
$(0,2)$	−	+	+	−	D
$(2,\infty)$	−	+	+	−	D

(f) local max: none

local min: $\boxed{x=0}$

(g) $f''(x) = \frac{-2(x^2-4)^2 - (-2x)(x^2-4)\cdot 2(2x)}{(x^2-4)^4}$

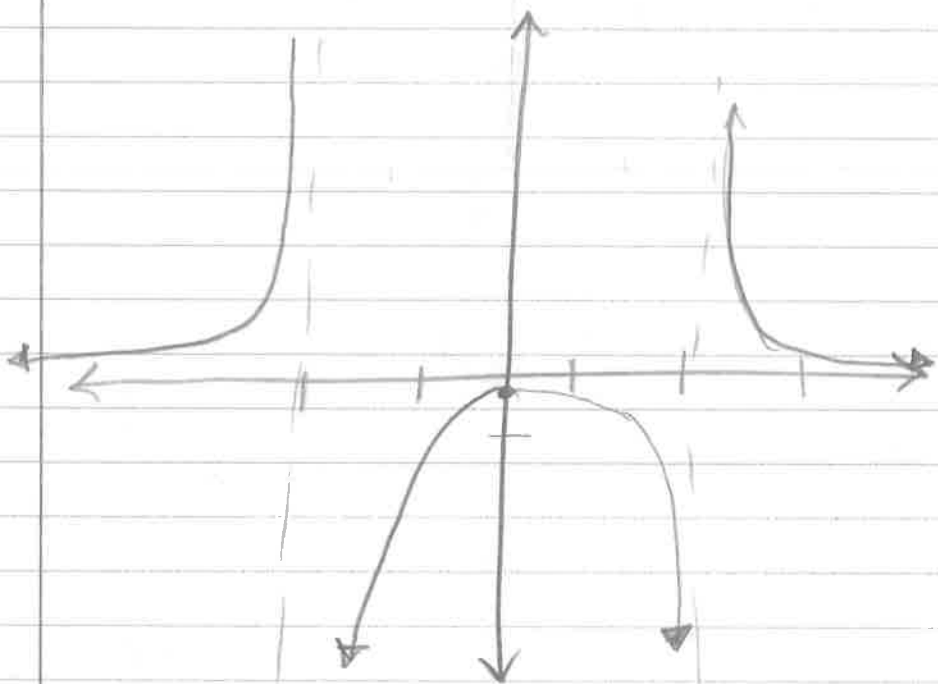
$= \frac{-2(x^4-8x^2+16)-(-8x^2)(x^2-4)}{(x^2-4)^4} = \frac{-2x^4+16x^2-32+8x^4-32x^2}{(x^2-4)^4}$

$= \frac{6x^4-16x^2-32}{(x^2-4)^4} = \frac{2(3x^4-8x^2-16)}{(x^2-4)^4} = \frac{2(3x^2+4)(x^2-4)}{(x^2-4)^{4\,3}}$

$$f''(x) = \frac{2(3x^2+4)}{(x^2-4)^3}$$

Intervals	$2(3x^2+4)$	$1/(x^2-4)^3$	$=f''$	Concavity
$(-\infty,-2)$	+	+	+	CU
$(-2,2)$	+	−	−	CD
$(2,\infty)$	+	+	+	CU

Inflection points: none



38

p.250 #22

$y=(x-4)\sqrt[3]{x}$

(a) $(-\infty,\infty)$

(b) $(0,0)$ $(4,0)$

(c) neither

(d) none

(e) $y' = -4(\sqrt[3]{x}) + (x-4)x^{-2/3} = -4\sqrt[3]{x} + \frac{x-4}{\sqrt[3]{x^2}}$

$4\sqrt[3]{x} = \frac{4x-4}{\sqrt[3]{x^2}}$

$0 = 4x-4$

$3x=-4$ $x=1$ or $x=0$

p.24

Interval	$f'(x)$	I/D
$(-\infty, 0)$	+	I
$(0, 1)$	−	D
$(1, \infty)$	+	I

8. local max: $x = 0$ local min: $x = 1$
9. $f''(x) = \dfrac{4(x+2)}{9x^{5/3}}$

Interval	$f''(x)$	Concavity
$(-\infty, -2)$	+	CU
$(-2, 0)$	−	CD
$(0, \infty)$	+	CU

inflection: $x = -2$ & $x = 0$