# Common Ground in Elementary Mathematics Education

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In a previous article, we analyzed some of the serious pitfalls in American elementary math education. However, this does not mean I hold a high opinion of the current state of math education in China either. In fact, I am equally critical of the widespread practice of inefficient, high-intensity drill-and-kill methods. Many students spend enormous amounts of time endlessly doing practice problems, yet they lack a systematic understanding of mathematics and proper logical training—this approach is both exhausting and ineffective.

In this article, I would like to walk through the essential content that constitutes **elementary mathematics**, and explain why we firmly believe that, if taught systematically, students can fully master this material within **four years**.

## The Basic Structure of Elementary Mathematics

Once the fundamental arithmetic operations (addition, subtraction, multiplication, division) and basic multiplication facts (such as the  $9\times9$  multiplication table) have been learned—typically by the 3rd or 4th grade—the core structure of elementary mathematics can be divided into three major parts:

- 1. Introductory Algebra
- 2. Geometry and Mathematical Proof
- 3. Precalculus

These arithmetic foundations usually conclude by the end of 4<sup>th</sup> grade. Students can then begin systematic algebra training. In this article, we will focus on the framework of *Introductory Algebra*. The frameworks of Geometry and Precalculus will be addressed in future writings. If you are interested, feel free to contact us by WeChat or email for a copy of our course syllabus.

# The Structure of Introductory Algebra

We begin with the most fundamental concepts: the natural numbers and the base-10 (decimal) number system. From understanding how numbers are represented and manipulated, students are naturally introduced to the three core algebraic operations: **addition**, **multiplication**, **and exponentiation**.

As students explore these operations, we guide them to discover and summarize the **three fundamental laws for addition and multiplication** (commutative, associative, and distributive

laws), along with the rules of exponentiation. These become the **axiomatic foundation** of algebra.

Most of these principles are expressed through equations, and we extend this framework by introducing the basic rules for equations and inequalities. Together, these become what we call the **"first principles"** of algebra. In our teaching, these are treated as axioms—foundational rules upon which all algebraic reasoning is built.

Once students have a solid grasp of these rules, we introduce the concept of **inverse operations**, helping them understand the essential nature of subtraction and division. This leads to the concepts of **opposites and reciprocals**, and the introduction of **negative numbers and fractions**. At this point, students have expanded their number system from the natural numbers to the **rational numbers**, completing the full scope of arithmetic operations and understanding.

Next, we generalize the base-10 system to **other number bases**, such as **binary (base-2)**, which deepens students' understanding of number representation. If we replace the base with a variable x, we are naturally led to the general concept of **polynomials**:

$$C_n x^n + C_{n-1} x^{n-1} + \dots + C_1 x + C_0.$$

The **central task** of Introductory Algebra is helping students master the **four basic operations of polynomials**. Interestingly, many students eventually find that polynomial operations are even more structured and easier to understand than arithmetic with base-10 numbers.

Following exponentiation, we naturally encounter its inverse—**roots**—and the need to **solve polynomial equations**, especially quadratic equations. To fully understand the solutions to quadratic equations, we expand the number system again to include **irrational and complex numbers**, and teach students how to perform the basic operations with complex numbers.

At this point, students are able to master and even prove the Fundamental Theorem of Algebra for quadratic equations.

To solve higher-degree equations—cubic, quartic, quintic, and beyond—students need to understand the **polar form of complex numbers** and **Euler's formula**. Since this requires knowledge of angles, similarity, and trigonometric functions, we will cover those topics in our **Geometry and Proof**, and **Precalculus** courses.

## The Design Philosophy of the "Everyone Math" Curriculum

#### Why Do We Advocate Completing Elementary Math in Four Years?

As previously mentioned, in the U.S., students typically begin Algebra I in 9th grade and reach Precalculus by 12th grade. That's already quite late—by then, there's almost no time left to explore other STEM courses.

In China, the situation is the opposite extreme. Although Chinese students begin learning math earlier, in reality, only **four out of six secondary school years** are truly devoted to studying elementary mathematics in a systematic way—two years in middle school and two in high school. The remaining time is largely consumed by exam preparation and endless drills.

Thus, we clearly propose: **students should complete the entire elementary mathematics curriculum between grades 6 and 10**. This paves the way for learning other STEM subjects such as physics, chemistry, and computer science. With a solid math foundation, students can understand and apply their knowledge more deeply, creating a virtuous cycle of reinforcement and higher-level learning.

This idea is not theoretical. It comes from more than a decade of teaching experience and working with students at all levels. It is also the core design principle behind the courses at **Everyone Academy**. We are confident that more and more families and students will recognize this and join us in building a more effective math education path.

### The Structure of Our Introductory Algebra Course

Our first course, Introductory Algebra, is designed with the following principles:

- No separation into Pre-Algebra, Algebra I, or Algebra II
  - We teach algebra as a complete and coherent course, avoiding unnecessary fragmentation.
- Recommended before adolescence
  - We encourage students to start this course before 7th grade. Parents are welcome to learn alongside their children.
- Option to skip the course via placement test
  - Students in 7th grade or above can take our placement test. Those who do well in the test may skip part of the course.
- Unlimited free re-enrollment
  - To ensure mastery, we promise that any student enrolled in the course may **retake it for free as many times as they wish**, until they have truly understood the material.
- Course format
  - o 32 weeks total
  - Each week: 1.5 hours of lecture + 0.5 hours of Q&A/practice
  - Q&A sessions are led by former students who have completed our "Geometry and Proof" course and passed our teaching assistant training.
- Currently offered in English only
  - Chinese-language instruction may be offered in the future, subject to changes in national policy.

If you want your child to build a **solid foundation in mathematics**, and not fall behind or miss opportunities, then **systematic and effective learning of algebra** is the essential first step. Avoid inefficient cramming and disjointed practice. We warmly welcome parents who take their children's education seriously to join us in this effort.

★ Learn more:

Everyone Academy Official Website

Let's work together to help children master this most fundamental—and most important—language of science: **mathematics**.