Repeating decimals- dangerous yet challenging topics

Meijun Zhu

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This article is suitable for everyone who took the 5th grade math. Hopefully, after 10 years from now (particularly after enough people read this article), we move the concept of repeating decimals out of any Algebra textbooks, but to textbooks for precalculus.

In the current years, ChatGPT is very hot. So I decide to ask it the first question:

$$\boldsymbol{Is\ \frac{1}{3} = 0.333333\ \ldots\ \ldots ?}$$

Here is the answer by ChatGPT 3.5:

Yes, that's correct. When you represent the fraction 1/3 as a decimal, it is equal to 0.333333... The decimal representation of 1/3 is a repeating decimal where the digit 3 repeats infinitely.

What can you expect from ChatGPT: all textbooks may give you quite unanimous answers. Repeating decimal: it is dangerous if we want to veto it! So, how about **we just suggest that we shall postpone introducing this concept after we teach students about infinite series?**

We show below: how much trouble (and how many misconceptions) if we introduce the repeating decimals in Algebra.

We shall show how we obtain the repeating decimal $\boldsymbol{\frac{1}{3} = 0.333333\ \ldots\ \ldots}$ (see the *Precalculus* book by the author).

A naïve arithmetic calculation leads us to the following confusing long division:

$$
\begin{array}{r}
0.3333 \\
\hline
3\,)\,1.0000 \\
-\;\;9 \\
\hline
\;\;10 \\
-\;\;90 \\
\hline
\;\;\;\;10 \\
-\;\;\;\;9 \\
\hline
\;\;\;\;\;\;1
\end{array}
$$

We thus have the following notation:

$$\frac{1}{3} = 0.\dot{3} = 0.333333\ldots\ldots$$

and call the right-hand side a "repeating decimal number".

We now take a closer inspection. The first step in the above calculation is fine:

$$\frac{1}{3} = 0.3 + \frac{0.1}{3}.$$

The next step is also okay:

$$\frac{1}{3} = 0.3 + 0.03 + \frac{0.01}{3}.$$

But who can guarantee that the computation can go on? Even if it can go on, what is the real meaning of this procedure? Do not blame pupils who get confused here.

On the other hand, since the division is the inverse operation of the multiplication,

$$\frac{1}{3} = 0.\dot{3} = 0.333333\ldots\ldots$$

is equivalent to

$$1 = 3 \times 0.333333 \ldots \ldots$$

So, we need to convince people that

$$3 \times 0.333333 \ldots \ldots = 0.99999 \ldots \ldots = 1.$$

Of course, the first step in the above calculation can NOT be properly explained to pupils since the distribution rule may not be applied to the infinite summation! The second step, which is often taught in an "advanced" math class, is the follows.

Let

$$x = 0.99999 \ldots \ldots,$$

then

$$10x = 9.99999 \ldots \ldots$$

(Now, you see that the distributive rule is misused again!) . And

$$9.99999 \ldots \ldots . = 9 + 0.99999 \ldots \ldots = 9 + x.$$

The associative rule is also misused in the first step!!! If you do not believe, you can try to figure out what is

$$1 - 1 + 1 - 1 + 1 - 1 + \cdots =?$$

Is it 1 (if you believe the above is the same as $1 + (-1 + 1) + (-1 + 1) + (-1 + 1) \ldots$)? Or is it 0 (if you believe the above is the same as $(1 - 1) + (1 - 1) + (1 - 1) + \cdots$)?

With all these potential problems, are you still interested in teaching pupils about the repeating decimals?

Let's wait until we teach them "infinite series" in a precalculus course!