Review for Midterm 1

First Midterm: Feb. 21. Extra office hour: Feb. 19, 2:30-4:30pm

On Infinite sequences and series (Chapter 11).

Infinite sequences: Limit, algebraic operations, definition of limit, monotone, bound, and .... the SQUEEZE!

Exercise 1: Find the limits of

(a).

$$\lim_{n\to+\infty} \frac{(lnn)^2}{\sqrt{n}},$$

(b).

$$\lim_{n\to+\infty} n^{\frac{1}{n}},$$

(c).

$$\lim_{m\to+\infty} \sqrt{m+1} - \sqrt{m}.$$

Series: The meaning of convergence of a series, tests for divergence and convergence.

1. Divergent test.

Exercise 2. Test the convergence of the series:

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{(\ln n)^{100}}.$$

2. convergent test.

2.1. Two standard series: geometric series and $p-$ series; Comparison (Small of large) for positive series.

Exercise 3: Test the convergence of the series:
(a).

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + \ln n},$$

(b).

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + (-1)^n n}.$$

2.2. Alternating series.

2.3. Absolutely convergent and conditionally convergent; ratio and root tests: comparison by using geometric series.

Exercise 4: Test the convergence of the series:
(a).

$$\sum_{n=1}^{\infty} \frac{n!}{n^{1000}},$$

(b).

$$\sum_{n=1}^{\infty} \frac{2^n + 1}{2^n + 3^n}.$$

2.4. The integral test.

Exercise 5: Test the convergence of the series:
a).

$$\sum_{n=1}^{\infty} \frac{1}{n \ln n},$$

(b).

$$\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}.$$

<u>Power series</u>: Radius of convergence, convergent interval; Taylor and Maclaurin series.

Exercise 6. Find the radius of convergence and the interval of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^{2n}(2n)!}.$$

Exercise 7. Find the Taylor series for given functions at given point.
(a).

$$f(x) = x^2 + 4x + 5, \quad at \;\; x = -2.$$

(b).

$$f(x) = e^{x^2}, \quad at \quad x = 0.$$

(c).

$$f(x) = \frac{1}{(1-x)^2}.$$

WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!

Comments and question to: mzhu@math.ou.edu