Review for final exam

Final is a cumulative exam.

Final exam: Dec. 15, 8:00-10:00am.

On Infinite sequences and series (Chapter 12).

Infinite sequences: Limit, algebraic operations, definition of limit, monotone, bound, the squeeze theorem and L'Hospital's rule!

Exercise 1: Find the limits of

(a).

$$\lim_{n\to+\infty} \frac{(lnn)^2}{\sqrt{n}},$$

(b).

$$\lim_{n\to+\infty} n^{\frac{1}{n}},$$

(c).

$$\lim_{m\to+\infty} \sqrt{m+1} - \sqrt{m}.$$

Exercise 2: What is wrong when you apply L'Hospital's rule in the following solution?

$$\lim_{x\to 0} \frac{x+1}{x+2} = \lim_{x\to 0} \frac{(x+1)'}{(x+2)'} = \lim_{x\to 0} 1 = 1.$$

Series: The meaning (or DEFINITION) of the convergence of a series. Tests for divergence and convergence: Two standard series (geometric series and $p-$ series), comparison for positive series ((Small or large, or the LIMIT Comparison); Monotonic series (alternating series, the integral test); Absolutely convergent and conditionally convergent; Ratio and root tests: comparison by using geometric series.

Exercise 3. Test the convergence of the series:

(a).

$$\sum_{n=1}^{\infty}(-1)^{n^2+1}\frac{n}{(\ln n)^5}.$$

(b).

$$\sum_{n=1}^{\infty}\frac{1}{n^2+\ln n},$$

(c).

$$\sum_{n=1}^{\infty}\frac{1}{n^2+(-1)^n n}.$$

(d).

$$\sum_{n=1}^{\infty}\frac{n!}{n^{1000}},$$

(e).

$$\sum_{n=1}^{\infty}\frac{2^n+1}{2^n+3^n}.$$

(f).

$$\sum_{n=1}^{\infty}\frac{1}{n\ln n},$$

(g).

$$\sum_{n=1}^{\infty}\frac{1}{n(\ln n)^2}.$$

Power series: Radius of convergence, convergent interval; Taylor and Maclaurin series; relation between a function and it's Maclaurin series (control the remainder term); Binomial series.

Exercise 4. Find the Taylor series for given functions at given point.

(a).

$$f(x) = x^2 + 4x + 5, \quad at \quad x = -2.$$

(b).

$$f(x) = \cos x^2, \quad at \quad x = 0.$$

(c).

$$f(x) = (2+x)^{-3}, \quad at \quad x = 0.$$

Exercise 5*. Try the following two questions:
(a). Prove that

$$e^2 = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \cdots = \sum_{n=0}^{\infty} \frac{2^n}{n!}.$$

(b). Find a sequence x_n such that $\lim_{n\to\infty}(x_n - x_{n-1}) = 0$, but $\lim_{n\to\infty} x_n = +\infty$.

On Parametric equations and Polar coordinates (Chapter 11).

<u>Parametric curves</u>: Parametric equations are more general form; Calculus on parametric equations; Area and Length

Exercise 6: Change the standard equation for ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

into a parametric equation, then find the area of the region enclosed by the ellipse.

<u>Polar coordinate systems</u>: Intersection point(s); Area and length formula

Exercise 7: (a). Find ALL intersection points: $r = 2$ and $r = 2\cos 2\theta$.

(b). Find the area of the region that lies inside both of the circles $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$.

On (Vectors and vector function and geometry of Spaces Chapter 13 and 14).

<u>Vectors and vector functions</u>: Algebraic operation, vector products (Dot and cross, and scalar triple); Geometric meanings (addition, subtraction, dot product, cross product and mixed product); Derivative and integral of vector functions; representations of line and plane in space.

Exercise 8. (a). Find

$$\mathbf{i} \times (\mathbf{j} \times \mathbf{k}) = ?$$

$$\mathbf{i} \times (\mathbf{i} \times \mathbf{k}) = ?$$

(b). Prove the parallelogram law:

$$|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2).$$

(c). Assume $\mathbf{a} \neq \mathbf{0}$. If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, is $\mathbf{b} = \mathbf{c}$? What conclusion can you have?

Exercise 9: (a). Do the following two line intersect each other? If yes, find the plane to contain them: $\mathbf{r} = (1, 2, 3) + t(2, 3, 1)$; $\mathbf{r} = (2, 2, 1) + t(1, 3, 1)$.

(b). Find the intersection points between $\mathbf{r}(t) = (\cos t, \sin t, t)$ and $\mathbf{r}(t) = ((1 + t), t^2, t^3)$. Can you define (in a natural way) the angle beween these two curves at the intersection points? Can you compute it?

<u>Curvature</u>: Arc length and re-parametrization; Tangent, normal and binormal vectors to a space curve; Curvature (the definition and formulas).

Exercise 10: Show that the curvature of a plane curve: $x = f(t)$, $y = g(t)$ is given

$$\kappa = \frac{|x'y'' - y'x''|}{[x'^2 + y'^2]^{3/2}}.$$

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Office hour for the final exam:

Wednesday 10:30am-2:30pm.