Review for Midterm 1

Office hour: September 26, 2:00-3:30pm; Exam: Oct. 1

I. Partial derivative

We start to learn functions of several variables.

Definition:
How to find **Domain**, what is **Level curves** (and **Level curves**, etc.

Limit and Continuity:
When an rational polynomial has **NO** limit: Find two difference paths.
How to find limit? Definition is **TOO HARD**. But if the function is continuous at the point, it is easy!

Try these examples
Exercise 1: If the limit exists, find it. If it does not exist, give the reason.
1).

$$\lim_{(x,y)\to(0,0)} \frac{x^{10} + y\ln(x^2+1)}{x^3 + y^6 + 1}.$$

2)

$$\lim_{(x,y)\to(0,0)} \ln(x^2 + y^3 + 2).$$

3)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8 + y^2}.$$

Partial derivative:

How to compute the partial derivatives (include how to check a function solving a partial differential equation): All those old tricks from previous Calculus courses are still useful. Key part: **Chain rule**.

Exercise 2: Check $u(x,t) = f(x+at)$ solves the following equation:

$$u_{tt} - a^2 u_{xx} = 0.$$

Applications of derivatives: Tangent plane and normal line. The general way to find the formula using level surface: Where the normal direction

$$(F_x(x_0,y_0,z_0), F_y(x_0,y_0,z_0), F_z(x_0,y_0,z_0))$$

come from?

Exercise 3: Assume that $g(x,y) = f(x^2 + \sin y)$ where $f(t)$ is a differentiable function of t. If $f'(0) = 2$,
(a). Find $\partial g/\partial y(0,0) = ?$.
(b). Find the tangent plane of $g(x,y)$ at point $(0,0)$.

Exercise 4:
Find the tangent plane and normal line to the sphere $x^2 + (y-1)^2 + z^2 = 2$ at point $(1,1,1)$.

Maximum and Minimum

How to find **Critical Points**: Solve the system of equations.

How to distinguish the local MAX and MIN from critical points: compute the determinant and f_{xx}.

How to find GLOBAL extreme value: 1). fine extreme value insider the region, 2). find with constraint using Lagrange multipliers.

Exercise 5. Typical example: find the maximum and minimum value of $f(x,y) = xy$ in the closed disk: $\{(x,y) \ : \ x^2 + y^2 \leq 1\}$.

II. Double integrals
We start with the definition of double integral (the **VOLUME**).

How to compute it?
Integrals on rectangles: The link between a double integral and **ITERATED ingeral** is the deep **FUBINI's** theorem.

Exercise 6: Evaluate

$$\int\int_R xye^{x+y^2} dA$$

where $R = [0,1] \times [0,1]$.

Integrals on general regions: Basically, we extend $f(x, y)$ to $F(x, y)$ over a bigger rectangle. **TWO TYPES** regions.

One:

$$D = \{(x, y) \ : \ a \leq x \leq b, \quad (\text{for} \quad \text{fixed} \quad \text{x}), \quad g_1(x) \leq y \leq g_2(x)\};$$

The second

$$D = \{(x, y) \ : \ c \leq y \leq d, \quad (\text{for} \quad \text{fixed} \quad \text{y}), \quad h_1(y) \leq x \leq h_2(y)\}.$$

Change the order of iterated integrals

Exercise 7: Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} dx dy.$$