

Review for Midterm 2

Office hour: October 28, 2:00-3:30pm; Exam: Oct. 29

I. Double integrals

We start with the definition of double integral (the **VOLUME**).

How to compute it?

Integrals on rectangles: The link between a double integral and **ITERATED ingeral** is the deep **FUBINI**'s theorem.

Exercise 1: Evaluate

$$\int \int_R xy e^{x+y^2} dA$$

where $R = [0, 1] \times [0, 1]$.

Integrals on general regions: Basically, we extend $f(x, y)$ to $F(x, y)$ over a bigger rectangle. **TWO TYPES** regions.

One:

$$D = \{(x, y) : a \leq x \leq b, \text{ (for fixed } x), g_1(x) \leq y \leq g_2(x)\};$$

The second

$$D = \{(x, y) : c \leq y \leq d, \text{ (for fixed } y), h_1(y) \leq x \leq h_2(y)\}.$$

Change the order of iterated integrals

Exercise 2: Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} dx dy.$$

Polar coordinates

$$\int \int_R \sqrt{1 + f_x^2 + f_y^2} dA = \int_a^b \int_{\rho_1(r)}^{\rho_2(r)} f(r \cos \theta, r \sin \theta) r d\theta dr.$$

Surface area

$$S = \int \int_R \sqrt{1 + f_x^2 + f_y^2} dA.$$

II. Triple integrals

The definition of triple integral (the **VOLUME** of a **FOUR** dimensional solid).

How to compute it?

Integrals on boxes $B = [a, b] \times [c, d] \times [r, s]$: **FUBINI's** theorem is used to reduce it to a **DOUBLE ingeral** first.

Integrals on a general region E : **Three TYPES** regions (and many many cases!).
One example:

$$E = \{(x, y, z) : (x, y) \in D, \text{ (for fixed (x,y))}, \phi_1(x, y) \leq z \leq \phi_2(x, y)\};$$

Then, remember to describe the region $D!!!$

Another example

$$E = \{(x, y, z) : (x, z) \in D, \text{ (for fixed (x,z))}, \phi_1(x, z) \leq y \leq \phi_2(x, z)\}.$$

AND DO NOT forget to describe the region $D!!!$

Exercise 3: Using triple integral to find the volume of the solid bounded by the cylinder $x = y^2$, and the planes $z = 0$ and $x + z = 1$.

III. Transformation

$$T : \begin{cases} x = x(u, v) \\ y = y(u, v) \end{cases}$$

is a transformation from region S to R , then

$$\int \int_R f(x, y) dA = \int \int_S f[x(u, v), y(u, v)] \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dA.$$

So, we have the formula of double integrals in polar coordinates, etc.

Exercise 4: Using polar coordinates to evaluate

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2}} dx dy.$$

Exercise 5: Evaluate

$$\int \int_R \cos\left(\frac{y-x}{y+x}\right) dA,$$

where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$ and $(0, 1)$.

Copyright by Meijun Zhu