Review for Midterm 2

Office hour: October 28, 2:00-3:30pm; Exam: Oct. 29

I. Double integrals

We start with the definition of double integral (the **VOLUME**).

How to compute it?

Integrals on rectangles: The link between a double integral and **ITERATED ingeral** is the deep **FUBINI's** theorem.

Exercise 1: Evaluate

$$\int\int_R xye^{x+y^2}dA$$

where $R = [0,1] \times [0,1]$.

Integrals on general regions: Basically, we extend $f(x,y)$ to $F(x,y)$ over a bigger rectangle. **TWO TYPES** regions.

One:

$$D = \{(x,y) \ : \ a \leq x \leq b, \quad (\text{for fixed x}), \quad g_1(x) \leq y \leq g_2(x)\};$$

The second

$$D = \{(x,y) \ : \ c \leq y \leq d, \quad (\text{for fixed y}), \quad h_1(y) \leq x \leq h_2(y)\}.$$

Change the order of iterated integrals

Exercise 2: Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}dxdy.$$

Polar coordinates

$$\int\int_R f(x,y)dA = \int_a^b \int_{\rho_1(r)}^{\rho_2(r)} f(r\cos\theta, r\sin\theta)rd\theta dr.$$

Surface area

$$S = \int\int_R \sqrt{1+f_x^2+f_y^2}\,dA.$$

II. Triple integrals

The definition of triple integral (the **VOLUME** of a **FOUR** dimensional solid).

How to compute it?

Integrals on boxes $B = [a, b] \times [c, d] \times [r, s]$: **FUBINI's** theorem is used to reduce it to a **DOUBLE ingeral** first.

Integrals on a general region E: **Three TYPES** regions (and many many cases!). One example:

$$E = \{(x, y, z)\ : (x, y) \in D, \quad \text{(for fixed (x,y))}, \quad \phi_1(x, y) \le z \le \phi_2(x)\};$$

Then, remember to describe the region D!!!

Another example

$$E = \{(x, y, z)\ :\ (x, z) \in D, \quad \text{(for fixed (x,z))}, \quad \phi_1(x, z) \le y \le \phi_2(x, z)\}.$$

AND DO NOT forget to describe the region D!!!

Exercise 3: Using triple integral to find the volume of the solid bounded by the cylinder $x = y^2$, and the planes $z = 0$ and $x + z = 1$.

III. Transformation

$$T: \quad \begin{cases} x = x(u, v) \\ y = y(u, v) \end{cases}$$

is a transformation from region S to R, then

$$\int\int_R f(x, y)dA = \int\int_S f[x(u, v), y(u, v)]\left|\frac{\partial(x, y)}{\partial(u, v)}\right|dudv.$$

So, we have the formula of double integrals in polar coordinates, etc.

Exercise 4: Using polar coordinates to evaluate

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2}}\,dxdy.$$

Exercise 5: Evaluate

$$\int\int_R \cos\left(\frac{y-x}{y+x}\right)dA,$$

where R is the trapezoidal region with vertices (1, 0), (2, 0), (0, 2) and (0, 1).