# Review for the final exam

**Next Tuesday class (December 3) is moved to next Friday afternoon 1:00–3:00pm at PHSC 1025**

Final examination time: December 10, 8:00am-10:00am.

Final examination is a cumulative test, which will cover Chapter 15 to Chapter 16 (up to section 16.7).

Office hour for final examination time: December 6, 10:30am-12:00pm;

For the complete review, please check the previous reviews. Here I just outline some materials we covered AFTER the second midterm.

**I. Line integrals**

We learned three types line integrals: 1). Line integral of a scalar function; 2). Line integral of a scalar function with respect to one variable; 3) Line integral of a vector function (Physical meaning!)

**How to compute it?**

Case 1: From the definition.

**Exercise 1:** Evaluate

$$\int_C xydx + ydy$$

where $C$ is the sine curve $y = \sin x$, $0 \leq x \leq \pi/2$.

Case 2: Using the potential function for a conservative vector field.

**Exercise 2:** Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = (\frac{x}{(x^2+y^2)^{\frac{3}{2}}}, \frac{y}{(x^2+y^2)^{\frac{3}{2}}},$$

and

(a). $C$ is the upper half circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$;

or
(b). $C$ is a curve from $(1, 0)$ to $(0, 1)$.

Case 3: For a **closed simple** curve, one may also try Green's theorem.
**Exercise 3:** Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = ((1 + xy)e^{xy} + y, x + e^y + x^2 e^{xy}),$$

and $C$ is the circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \leq t \leq 2\pi$;

**II. Vector fields**

More attention on the conservative vector fields.

The ways to determine whether a vector field is conservative or not: Definition, Theorem 4 on Page 1129 (very abstract theorem), Theorem 6 for two dimensional vector field on page 1131.

Two big theorems we have learned:
**Fundamental theorem for line integral**, and the applications. (e.g. find potential functions).

**Green's theorem**, and its application. (e.g. Compute the line integrals).

**Exercise 4.** Let $f(x, y) = \sin(x - 2y)$ and $\mathbf{F} = \nabla f$. Can you find two simple curves $C_1$ and $C_2$ which are not closed, such that

$$(a) \quad \int_{C_1} \mathbf{F} d\mathbf{r} = 0, \qquad (b) \quad \int_{C_2} \mathbf{F} d\mathbf{r} = 1?$$

Curl vector and Divergence.
**Exercise 5.** Show that

$$curl(f\mathbf{F}) = f \cdot curl\mathbf{F} + \nabla f \times \mathbf{F}.$$

**III. Surface integrals and differential operators**

Parametric equations for a given surface. Surface areas. Tangent plane.
**Exercise 6:** Find the unit out normal vector for the surface $x^2 + y^2 - z = 0$ at $(2, 0, 4)$.

**Exercise 7.** Find the surface area of the surface $x^2 + z^2 = a^2$ that lies inside $x^2 + y^2 = a^2$.

Surface integrals: 1). surface integral of a scalar function. 2) Surface integral of a vector function.

**Exercise 8.** Find
$$\int\int_S (x^2 z + y^2 z) dS$$
where $S$ is the hemisphere $x^2 + y^2 + z^2 = 1,\ z \geq 0$.