# Review for the final exam

Final examination time: May 6, 1:30am-3:30am.

Final examination is a cumulative test, which will cover Chapter 14 to Chapter 16 (up to section 16.5).

Office hour for final examination time: May 2, 1:30pm-3:00pm;

For the complete review, please check the previous reviews. Here I just outline some materials we covered AFTER the second midterm.

**I. Line integrals**

We learned three types line integrals: 1). Line integral of a scalar function; 2). Line integral of a scalar function with respect to one variable; 3) Line integral of a vector function (Physical meaning!)

**How to compute it?**

Case 1: From the definition.

**Exercise 1:** Evaluate

$$\int_C xydx + ydy$$

where $C$ is the sine curve $y = \sin x$, $0 \leq x \leq \pi/2$.

Case 2: Using the potential function for a conservative vector field.

**Exercise 2:** Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = (\frac{x}{(x^2+y^2)^{\frac{3}{2}}}, \frac{y}{(x^2+y^2)^{\frac{3}{2}}},$$

and

(a). $C$ is the upper half circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$;

or

(b). $C$ is a curve from $(1, 0)$ to $(0, 1)$.

Case 3: For a **closed simple** curve, one may also try Green's theorem.

**Exercise 3:** Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = ((1+xy)e^{xy} + y, x + e^y + x^2 e^{xy}),$$

and $C$ is the circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \le t \le 2\pi$;

**II. Vector fields**

More attention on the conservative vector fields.

The ways to determine whether a vector field is conservative or not: Definition, Theorem 4 on Page 1129 (very abstract theorem), Theorem 6 for two dimensional vector field on page 1131.

Two big theorems we have learned:

**Fundamental theorem for line integral**, and the applications. (e.g. find potential functions).

**Green's theorem**, and its application. (e.g. Compute the line integrals).

**Exercise 4.** Let $f(x, y) = \sin(x - 2y)$ and $\mathbf{F} = \nabla f$. Can you find two simple curves $C_1$ and $C_2$ which are not closed, such that

$$(a) \quad \int_{C_1} \mathbf{F} d\mathbf{r} = 0, \qquad (b) \quad \int_{C_2} \mathbf{F} d\mathbf{r} = 1?$$

Curl vector and Divergence.

**Exercise 5.** Show that

$$curl(f\mathbf{F}) = f \cdot curl\mathbf{F} + \nabla f \times \mathbf{F}.$$