Review for Midterm 1

Midterm 1 on Feb. 26; Extra office hour: Feb. 21, 2:30–5:30pm

On inverse functions (Chapter 6)

Basic properties about inverse functions: One-one function, symmetric in graphs, relation between derivatives, etc.

Exercise 1: If $f(x) = \sqrt{x^3 + x^2 + x + 1}$, find $(f^{-1})'(2) =$.

Logarithmic and exponential functions: The way we introduce these concepts: Natural Logarithmic function: $y = \ln x = \int_1^x \frac{1}{t} dt$, $\Rightarrow$ its inverse natural exponential function: $y = e^x$, $\Rightarrow$ general exponential function: $y = a^x$ (what is the meaning of $2^{\sqrt{2}}$?), $\Rightarrow$ general logarithmic function: $y = log_a x$.

Exercise 2:

(a) Find the derivative of $f(x) = \ln \frac{1}{1+x^2}$

(b). What is the derivative of $y = x^{x+1}$? What is the domain of the function?

(c). Find

$$\int_e^{e^3} \frac{\ln x}{x} dx =$$

$sin^{-1}x$ and $cos^{-1}x$: Domain and range; derivative.

Exercise 3:

(a). Find $\sin[\cos^{-1} \frac{1}{2}]$ and $\tan[\sin^{-1} \frac{1}{3}]$.

(b). Show that

$$\frac{d(arcsinx)}{dx} + \frac{d(arccosx)}{dx} = 0.$$

(c). Show that

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}.$$

(d). Can you find:

$$tan^{-1}x + ctan^{-1}x = ?$$

L'Hospital's rule: **ALWAYS CHECK CONDITIONS BEFORE YOU USE THE RULE!**

Exercise 4:
(a) Find what is wrong:

$$\lim_{x\to 1}\frac{x^2}{x+1}=\lim_{x\to 1}\frac{2x}{1}=2.$$

(b). Find

$$\lim_{x\to +\infty}\frac{(\ln x)^3}{x}.$$

(c). Find

$$\lim_{x\to +0}\frac{(\ln x)^3}{x^{10}}.$$

(d). Find

$$\lim_{x\to +0}(1+\frac{1}{x})^{4x}.$$

(e). Find

$$\lim_{x\to +\infty}\frac{\sqrt{x^2+1}+10}{\sqrt[3]{8x^3-1000}}= .$$

Techniques of integration (Chapter 7).

Integration by parts 7.1: Choose your u and v. Sometimes, you need to integrate by parts twice.

Exercise 5: Find the integrals of the following

$$(a)\int x(\ln x)^2dx,\quad (b)\quad \int xe^{2x}dx,\quad (c)\int x\sin x dx,\quad (d)\int e^x\sin x dx.$$

Trigonometric integrals: Three types of integrals.
Exercise 6: Find the integrals of the following

$$(a)\int \sin^3 x dx,\quad (b)\quad \int \cos^2 x dx,\quad (c)\int \cos 2x\sin x dx.$$

Trigonometric substitution: $x=a\sin\theta$, $x=a\tan\theta$, etc. Remember to choose θ in a convenient range (for example: $\theta\in[-\pi/2,\pi/2]$.
Exercise 7: Find the integrals of the following

(a)

$$\int \frac{x^3}{\sqrt{1-x^2}}dx.$$

(b)

$$\int \frac{x^2}{\sqrt{1-x^2}}dx.$$

(c)

$$\int \frac{x+1}{1+x^2}dx.$$

Partial fractions: First, simplify the integrand (improper fraction → proper fraction). Then factorize the denominator.

Exercise 8: Find the integrals of the following

(a).

$$\int \frac{x+2}{4x^2+3x-1}dx.$$

(b).

$$\int \frac{1-2e^x}{1-e^{2x}}dx.$$

Strategy Sometimes you need to combine some techniques together, like in exercise 4 (b). One more example

Exercise 9: Find

$$\int e^{\sqrt{x}+1}dx.$$

Improper integrals Meanings, computations, comparison for convergence and divergence checking.

Exercise 10: Evaluate, if possible the improper integral

$$\int_1^3 \frac{1}{|x-2|^p}dx,$$

where p is a positive constant.

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