Review for Midterm 1

Office hour: September 21, 3:00-4:30pm; Exam: September 26

I: On (Vectors and vector function and geometry of Spaces Chapter 13 and 14).

Vectors and vector functions: Algebraic operation, vector products (Dot and cross, and scalar triple); Geometric meanings (addition, subtraction, dot product, cross product and mixed product); Derivative and integral of vector functions; representations of line and plane in space.

Exercise 1. (a). Find value x so that vectors $\mathbf{a} = (1, x, -2)$, $\mathbf{b} = (2, 0, -1)$ and $\mathbf{c} = (1, 1, -1)$ are on the same plane.

(b). Are the following four points on the same plane: A (1, 3, 5), B(1, 0, 2), C(3, 2, 1) and D(2, 1, 1)?

Exercise 2: (a). Do the following two line intersect each other? If yes, find the plane that contains them: $\mathbf{r} = (1, 2, 3) + t(2, 3, 1)$; $\mathbf{r} = (2, 2, 1) + t(1, 3, 1)$.

(b). Find the intersection points between $\mathbf{r}(t) = (\cos t, \sin t, t)$ and $\mathbf{r}(t) = ((1 + t), t^2, t^3)$. Can you define (in a natural way) the angle beween these two curves at the intersection points? Can you compute it?

Exercise 3: (a). Show that the distance between the given parallel planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$ is

$$distance = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

(b). Find the equations of the planes which is parallel to $x + 2y + 3z = 1$ and have distance 3 to this plane.

Curvature: Arc length and re-parametrization; Tangent to a space curve; Curvature (the definition and formulas).

Exercise 4: (a). Find the curvature at $t = \pi$ for $x = \cos t,\ y = 2 \sin t$.

(b).Show that the curvature of a plane curve: $x = f(t),\ y = g(t)$ is given

$$\kappa = \frac{|x'y'' - y'x''|}{[x'^2 + y'^2]^{3/2}}.$$

II. Partial derivative

We start to learn functions of several variables.

Definition:

How to find **Domain**, what are **Level curves** (and **Level surfaces**), etc.

Limit and Continuity:

When a rational polynomial has **NO** limit: Find two difference paths.

How to find limit? Definition is **TOO HARD**. But if the function is continuous at the point, it is easy!

Try these examples

Exercise 5: If the limit exists, find it. If it does not exist, give the reason.

1).

$$\lim_{(x,y)\to(0,0)} \frac{x^{10} + y\ln(x^2+1)}{x^3 + y^6 + 1}.$$

2)

$$\lim_{(x,y)\to(0,0)} \ln(x^2 + y^3 + 2).$$

3)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8 + y^2}.$$

Partial derivative:

How to compute the partial derivatives (include how to check a function solving a partial differential equation): All those old tricks from previous Calculus courses are still useful. Key part: **Chain rule**.

Exercise 6: Check that $u(x,t) = f(x + at)$ solves the following equation:

$$u_{tt} - a^2 u_{xx} = 0.$$

Applications of derivatives: Tangent plane and normal line. The general way to find the formula using level surface: Where does the normal direction

$$(F_x(x_0, y_0, z_0), F_y(x_0, y_0, z_0), F_z(x_0, y_0, z_0))$$

come from?

Exercise 7: Assume that $g(x, y) = f(x^2 + \sin y)$ where $f(t)$ is a differentiable function of t. If $f'(0) = 2$,

(a). Find $\partial g/\partial y(0, 0) =$?.

(b). Find the tangent plane of $g(x, y)$ at point $(0, 0)$.

Exercise 8:

Find the tangent plane and normal line to the sphere $x^2 + (y - 1)^2 + z^2 = 2$ at point $(1, 1, 1)$.