Review for Midterm 1— Oct. 3rd, 2023

Office hour: September 28, 10:20am-12:20pm

I. Partial derivative

We start to learn functions of several variables.

Definition:
How to find **Domain**, what are **Level curves** (and **Level surfaces**), etc.

Limit and Continuity:
When a rational polynomial has **NO** limit: Find two difference paths.
How to find limit? Definition is **TOO HARD**. But if the function is continuous at the point, it is easy!

Try these examples
Exercise 1: If the limit exists, find it. If it does not exist, give the reason.
1).

$$\lim_{(x,y)\to(0,0)} \frac{x^{10} + y\ln(x^2+1)}{x^3+y^6+1}.$$

2)

$$\lim_{(x,y)\to(0,0)} \ln(x^2+y^3+2).$$

3)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8+y^2}.$$

4) (Hard one)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y^2}{x^4+y^4}.$$

5)(Hardest one)

$$\lim_{(x,y)\to(0,0)} \frac{x^6y^4}{x^8+y^8}.$$

<u>Partial derivative</u>:

How to compute the partial derivatives (include how to check a function solving a partial differential equation): All those old tricks from previous Calculus courses are still useful. Key part: **Chain rule**.

Exercise 2: Check that $u(x,t) = f(x+at)$ solves the following equation:

$$u_{tt} - a^2u_{xx} = 0.$$

Applications of derivatives: Tangent plane and normal line. The general way to find the formula using level surface: Where does the normal direction

$$(F_x(x_0,y_0,z_0), F_y(x_0,y_0,z_0), F_z(x_0,y_0,z_0))$$

come from?

Exercise 3: Assume that $g(x,y) = f(x^2+\sin y)$ where $f(t)$ is a differentiable function of t. If $f'(0) = 2$,

(a). Find $\partial g/\partial y(0,0) =?$.

(b). Find the tangent plane of $g(x,y)$ at point $(0,0)$.

Exercise 4:

Find the tangent plane and normal line to the sphere $x^2+(y-1)^2+z^2 = 2$ at point $(1,1,1)$.

<u>Maximum and Minimum</u>

How to find **Critical Points**: Solve the system of equations.

How to distinguish the local MAX and MIN from critical points: compute the determinant and f_{xx}.

How to find GLOBAL extreme value: 1). fine extreme value insider the region, 2). find with constraint using Lagrange multipliers.

Exercise 5. Typical example: find the maximum and minimum value of $f(x,y) = xy$ in the closed disk: $\{(x,y) \ : \ x^2+y^2 \leq 1\}$.