# Review for Midterm — Oct. 13, 2025

**Office hour: Oct. 9, 2:30pm-3:0pm**

## I. Partial derivative

We start to learn functions of several variables.

Definition:
How to find **Domain**, what are **Level curves** (and **Level surfaces**), etc.

Limit and Continuity:
When a rational polynomial has **NO** limit: Find two difference paths.

How to find limit? Definition is **TOO HARD**. But if the function is continuous at the point, it is easy!

Try these examples

**Exercise 1**: If the limit exists, find it. If it does not exist, give the reason.

1).

$$\lim_{(x,y)\to(0,0)} \frac{x^{10} + y\ln(x^2+1)}{x^3+y^6+1}.$$

2)

$$\lim_{(x,y)\to(0,0)} \ln(x^2+y^3+2).$$

3)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8+y^2}.$$

4) (Hard one)

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y^2}{x^4+y^4}.$$

5)(Hardest one)

$$\lim_{(x,y)\to(0,0)} \frac{x^6y^4}{x^8+y^8}.$$

Partial derivative:

How to compute the partial derivatives (include how to check a function solving a partial differential equation): All those old tricks from previous Calculus courses are still useful. Key part: **Chain rule**.

**Exercise 2:** Check that $u(x,t) = f(x-at)$ solves the following equations:

$$u_t + au_x = 0$$

and

$$u_{tt} + a^2u_{xx} = 0.$$

Applications of derivatives: Tangent plane and normal line. The general way to find the formula using level surface: Where does the normal direction

$$(F_x(x_0,y_0,z_0), F_y(x_0,y_0,z_0), F_z(x_0,y_0,z_0))$$

come from?

**Exercise 3:** Assume that $g(x,y) = f(x^2+\sin y)$ where $f(t)$ is a differentiable function of $t$. If $f'(0) = 2$ and $f(0) = 3$,

(a). Find $\partial g/\partial y(0,0) =?$.

(b). Find the tangent plane of $g(x,y)$ at point $(0,0)$.

**Exercise 4:**

Find the tangent plane and normal line to the sphere $x^2+(y-1)^2+z^2 = 2$ at point $(1,1,1)$.

Maximum and Minimum

How to find **Critical Points**: Solve the system of equations.

How to distinguish the local MAX and MIN from critical points: compute the determinant and $f_{xx}$.

How to find GLOBAL extreme value: 1). fine extreme value insider the region, 2). find with constraint using Lagrange multipliers.

**Exercise 5**. Typical example: find the maximum and minimum value of $f(x,y) = \sqrt{x^2+2y^2+1}$ in the closed disk: $\{(x,y) \ : \ x^2+y^2 \leq 1\}$.

## II. Double integrals

We start with the definition of double integral (the **VOLUME**).

**How to compute it?**

Integrals on rectangles: The link between a double integral and **ITERATED ingeral** is the deep **FUBINI**'s theorem.

**Exercise 6:** Evaluate

$$\int\int_R xye^{x+y^2}dA$$

where $R = [0,1] \times [0,1]$.