

Review after second midterm exam

Final exam, December 15, 8:00am-10:00am.

Office hour: December. 14, 2:30pm-4:40pm;

I. Line integrals

We have learned two types line integrals: 1). Line integral of a scalar function (geometric meaning); 2). Line integral of a scalar function with respect to one variable.

How to compute it?

Case 1: From the definition.

Exercise 1: Evaluate

$$\int_C xy dx + y dy$$

where C is the sine curve $y = \sin x$, $0 \leq x \leq \pi/2$.

Case 2: Using the potential function for a conservative vector field.

Exercise 2: Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = \left(\frac{x}{(x^2 + y^2)^{\frac{3}{2}}}, \frac{y}{(x^2 + y^2)^{\frac{3}{2}}} \right),$$

and

(a) C is the upper half circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$;

or

(b) C is a curve from $(1, 0)$ to $(0, 1)$.

Case 3: For a **closed simple** curve, one may also try Green's theorem.

Exercise 3: Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = ((1 + xy)e^{xy} + y, x + e^y + x^2e^{xy}),$$

and C is the circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \leq t \leq 2\pi$;

II. Vector fields

More attention on the conservative vector fields.

The ways to determine whether a vector field is conservative or not: Definition, theorem 4 on Page 1129 (we called it the **useless** theorem), Theorem 6 for two dimensional vector field on page 1131.

Two big theorems we have learned:

Fundamental theorem for line integral, and the applications. (e.g. find potential functions).

Green's theorem, and its application. (e.g. Compute the line integrals).

Exercise 4. Let $f(x, y) = \sin(x - 2y)$ and $\mathbf{F} = \nabla f$. Can you find two simple curves C_1 and C_2 which are not closed, such that

$$(a) \int_{C_1} \mathbf{F} d\mathbf{r} = 0, \quad (b) \int_{C_2} \mathbf{F} d\mathbf{r} = 1?$$

III. Surface integrals and differential operators

Parametric equations for a given surface. Surface areas. Tangent plane.

Exercise 5: Find the unit out normal vector for the surface $x^2 + y^2 - z = 0$ at $(2, 0, 4)$.

Exercise 6. Find a parametric representative for part of the sphere $x^2 + y^2 + z^2 = 36$ that lies between the planes $z = 0$ and $z = 3\sqrt{3}$.

Surface integrals: 1). surface integral of a scalar function. 2) Surface integral of a vector function.

Curl vector and Divergence.

Exercise 7. Show that

$$\nabla(u \nabla v) = \nabla u \cdot \nabla v + u \Delta v,$$

where $\Delta v = v_{xx} + v_{yy} + v_{zz} = \text{div}(\text{grad} v)$.

Exercise 8. Find

$$\int \int_S (x^2 z + y^2 z) dS$$

where S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

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