# Review after second midterm exam

Final exam, December 11, 8:00am-10:00am.

Office hour: December 8, 1:30pm-3:40pm;

**I. Line integrals**

We have learned two types line integrals: 1). Line integral of a scalar function (geometric meaning); 2). Line integral of a scalar function with respect to one variable.

**How to compute it?**

Case 1: From the definition.

**Exercise 1:** Evaluate

$$\int_C xydx + ydy$$

where $C$ is the sine curve $y = \sin x$, $0 \leq x \leq \pi/2$.

Case 2: Using the potential function for a conservative vector field.

**Exercise 2:** Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = (\frac{x}{(x^2+y^2)^{\frac{3}{2}}}, \frac{y}{(x^2+y^2)^{\frac{3}{2}}}),$$

and

(a) $C$ is the upper half circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$;

or

(b) $C$ is a curve from $(1, 0)$ to $(0, 1)$.

Case 3: For a **closed simple** curve, one may also try Green's theorem.

**Exercise 3:** Evaluate

$$\int_C \mathbf{F} d\mathbf{r}$$

where

$$\mathbf{F} = ((1+xy)e^{xy} + y, x + e^y + x^2 e^{xy}),$$

and $C$ is the circle: $\mathbf{r}(t) = (\cos t, \sin t)$, $0 \le t \le 2\pi$;

## II. Vector fields

More attention on the conservative vector fields.

The ways to determine whether a vector field is conservative or not: Definition, theorem 4 on Page 1129 (we called it the **useless** theorem), Theorem 6 for two dimensional vector field on page 1131.

Two big theorems we have learned:

**Fundamental theorem for line integral**, and the applications. (e.g. find potential functions).

**Green's theorem**, and its application. (e.g. Compute the line integrals).

**Exercise 4.** Let $f(x,y) = \sin(x-2y)$ and $\mathbf{F} = \nabla f$. Can you find two simple curves $C_1$ and $C_2$ which are not closed, such that

$$(a) \quad \int_{C_1} \mathbf{F} d\mathbf{r} = 0, \qquad (b) \quad \int_{C_2} \mathbf{F} d\mathbf{r} = 1?$$

## III. Differential operators

Curl vector and Divergence.

**Exercise 5.** (1) Verify vector field $\mathbf{F} = (e^{yz}, xze^{yz}, xye^{yz})$ is conservative.
(2) Find one potential function $f(x,y,z)$ for $\mathbf{F}$

**Exercise 6.** Let $u = u(x,u,z)$, $v = v(x,y,z)$. Show that

$$\nabla(u\nabla v) = \nabla u \cdot \nabla v + u\Delta v,$$

where $\Delta v = v_{xx} + v_{yy} + v_{zz} = div(grad\ v)$.