Review for Midterm 1

First order ODEs

Definition: We start with some terminologies: Differential equations, solutions, general solutions, particular solutions, etc

Exercise 1: Check $y = e^{-2x}$ solves

$$y" - 4y = 0.$$

How to solve a differential equation (1st order): 7 types:

Type 0:

$$\frac{dy}{dx} = f(x).$$

We learned it in Calculus course.

Type 1:

$$\frac{dy}{dx} + P(x)y = Q(x).$$

Linear. Find an integrating factor.

Exercise 2: Solve

$$\frac{dy}{dx} + xy = x^2.$$

Type 2:

$$\frac{dy}{dx} = g(x)h(y).$$

Separable equations! Change it to

$$\frac{dy}{h(y)} = g(x)dx.$$

Caution: $h(y) = 0$ **might be a solution!**

Exercise 3: Solve

$$\frac{dy}{dx} + x = xy^2.$$

Type 3:

$$\frac{dy}{dx} = F(ax + by + c).$$

Linear substitution: let $v(x) = ax + by + c$.

Exercise 4: Solve

$$\frac{dy}{dx} = (x + y + 2)^2.$$

Type 4:

$$\frac{dy}{dx} = F(\frac{y}{x}).$$

Nonlinear substitution: let $v(x) = y/x$.

Exercise 5: Solve

$$2xy\frac{dy}{dx} = 4x^2 + 3y^2.$$

Type 5:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Bernoulli equations. Change it to

$$\frac{dy}{y^n dx} + P(x)y^{-(n-1)} = Q(x).$$

Then, for $n \neq 1$, let $v(x) = y^{-(n-1)}$.

Exercise 6: Solve

$$y^2\frac{dy}{dx} + 2xy^3 = 6x.$$

Type 6:

$$M(x, y)dx + N(x, y)dy = 0.$$

Exact equations.

Exercise 7: Solve

$$(x^3 + \frac{y}{x})dx + (y^2 + lnx)dy = 0.$$

Other types (and they are hard!): $F(x, y', y'') = 0$; $F(y, y', y") = 0$.