Review for Midterm 2

Midterm 2, March 14

Linear ODEs

Definition: We start with some terminologies: Linear differential equations, superposition, general solutions, homogeneous and nonhomogeneous equations, particular solutions, etc

Exercise 1: If y_1 and y_2 solve equation

$$y^{''} + e^x y' + y \sin x^2 = 0,$$

check $2y_1 + 3y_2$ is also a solution.

How to solve a homogeneous equation of constant coefficients:

$$y^{(n)} + a_{n-1}y^{(n-1)} + ... + a_1 y' + a_0 y = 0.$$

Three types.

Consider the Characteristic equation.

Type 1: Distinct roots $R_1, ..., R_n$

Type 2: Repeat roots (roots with multiplicity greater than 1)

Exercise 2: Find and check the general solution to

$$y" + 4y' + 4y = 0.$$

Type 3: complex roots.

Exercise 3: Find the general solutions , if the characteristic equation of a ODE is

(a).

$$(r-1)^3(r^2 - 4r + 5) = 0,$$

or
(b).

$$(r-1)^3(r^2-2r+2)^2=0,$$

How to solve a nonhomogeneous equation of constant coefficients:

$$y^{(n)}+a_{n-1}y^{(n-1)}+...+a_1y'+a_0y=f(x).$$

Undetermined coefficient method:
Exercise 4: Find the general solution to

$$y"+4y'+5y=x+e^{2x}\sin x.$$

Variation of parameter method: the formula:

$$y(x)=-y_1\int\frac{y_2f(x)}{W(y_1,y_2)}dx+y_2\int\frac{y_1f(x)}{W(y_1,y_2})dx.$$

Exercise 5: Find the general solution to

$$y"+y=\frac{1}{\cos x}.$$

Variation of parameter method: the idea:***
Exercise 6: Check $y_1(x)=x^3$ is a solution to

$$x^2y''+xy'-9y=0$$

for $x>0$. Find the second solution that is linearly independent to $y_1(x)$.