

Review after Midterm Exam 2

Exam day: May 7. 1:30—3:30pm

Office hour: May 2, 11:30 am–1:15 pm

Eigenvalue problems

Exercise 1: Find all positive eigenvalues of

$$-y'' = \lambda y, \quad y'(0) = y'(\pi) = 0.$$

Algebraic methods

Laplace transform: Definition and basic formulas for e^{at} , $\cos kt$, $\sin kt$, t^a , translation and partial fraction, integral, differentiation and convolution

Exercise 2: Find: $\mathcal{L}^{-1}\{F(s)\}$ if

(a). $F(s) = \frac{1}{s(s-3)}$;

(b). $F(s) = \frac{1}{s^2(s-3)}$;

(c). $F(s) = \frac{s-1}{(s+1)^3} + \frac{1}{s(s^2+1)}$.

(d). $F(s) = \frac{s}{(s^2+1)^2}$.

Using Laplace transform to solve initial value problem: D.E to A.E., solving A.E., then find $\mathcal{L}^{-1}\{F(s)\}$.

Exercise 3: Solve:

(a). $x'' + 4x = \sin 2t$, $x(0) = x'(0) = 0$.

(b). $x' = 4x + 2y$, $y' = 3x - y$; $x(0) = 3$, $y(0) = -2$;

(c). $tx'' - 2x' + tx = 0$, $x(0) = 0$.

(d). $tx'' - 2x' + tx = 0$, $x(0) = 1$.

(f)* Can you find the general solution to $tx'' - 2x' + tx = 0$?

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