

Review for Midterm 2

Midterm 2: 10-30-2008. Will cover 4.1-4.2, 7.1-7.3, 8.1-8.2

Extra office hour: Wed: 2:30-4:00pm

2x2 Linear system

How to solve: 1). Elimination method (section 4.2); 2). Laplace transform method (section 7.2).

Exercise: 1). Using elimination method to find general solution to:

$$x' = x + 2y, \quad y' = x.$$

2). Using Laplace transform to solve:

$$x' = x + 2y, \quad y' = x + e^{-t}; \quad x(0) = y(0) = 0.$$

Can you use the elimination method to solve above problem?

Laplace transform

Definition: We start with Laplace transform (section 7.1). The basic algebraic properties about the Laplace transform: linearity, translation.

Exercise 2:

(a). If $L(f(t)) = F(p)$, show that

$$L(f(t) + 2e^{-3t}) = F(p) + \frac{2}{p+3}.$$

(b). $L(f(t)) = F(p)$, show from the definition that

$$L\{e^{-2t}f(t)\} = F(p+2).$$

How to solve an equation of constant coefficients: Transformation of initial value problems.

Step 1: Solve $L\{x(t)\} = X(s)$ algebraically.

Exercise 3:

(a). If $x(t)$ solves

$$x'' + 4x' + 13x = 20e^{-t}, \quad x(0) = 1, \quad x'(0) = 3.$$

Find

$$X(s) = L\{x(t)\}.$$

(b). If $x(t)$, $y(t)$ solve

$$x' + 2y = 1, \quad 2x - y' = 2t, \quad x(0) = 0, \quad y(0) = 1.$$

Find

$$X = L\{x(t)\} \quad \text{and} \quad Y = L\{y(t)\}.$$

Step 2: Find $x(t)$ from X :

Exercise 4:

(a). If

$$X = \frac{s}{s^2 + 3},$$

find $x(t)$ so that $L\{x(t)\} = X$.

(b). If

$$X = \frac{s + 4}{(s + 4)^2 + 3},$$

find $x(t)$ so that $L\{x(t)\} = X$.

Power series to solve ODEs: how to find the recursive relation.

Exercise 5 Using power series $y = \sum_{n=0}^{\infty} c_n(x-1)^n$ to solve

$$xy' + y = 0; \quad y(1) = 2.$$

(a). Find the recursive relation.

(b). Find the power series solution.

(c). Find the simple form of the solution, and find $y(\frac{3}{2})$.

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