Review for the course

Final Exam: 12-17-2008, 1:30pm-3:30pm. Will cover 1.1, 1.2, 1.3-1.6, 3.1-3.3, 3.5, 4.1, 4.2, 7.1-7.3, 8.1, 8.2, 9.1-9.3, 9.5-9.7.

Extra office hour: Monday December 15, 10:30am-2:30pm

Reviews for materials before Chapter 9 can be found in Review #1 and #2.

Trigonometric functions and Fourier series, 9.1-9.3.

Trigonometric functions: Period, inner product, orthogonality.

Exercise 1: Find period for functions:

(a). $f(x) = 2\sin(2x) + 4$; (b). $f(x) = 3\sin x - 4\cos(3x)$.

Exercise 2 (a). Show that if $m \neq n$,

$$\int_{-L}^{L} \sin \frac{m\pi}{L} x \sin \frac{n\pi}{L} x dx = 0.$$

(b). Find

$$\int_{-\pi}^{\pi} e^{2xi} \cdot e^{-3xi} dx$$

Fourier series: Fourier series in $[-L, L]$, sine and cosine series for function $f(x)$ defined on $[0, L]$.

Exercise 3:

(a). Find Fourier series for $f(t) = \frac{t}{2}, \quad -\pi < t < \pi$. This one is closely related to Question 19 in section 9.2 (page 595).

(b). Find the sine series for $f(t) = t$ defined on $(0, \pi)$.

Solving partial differential equations using Fourier series, 9.5-9.7.

Eigenvalue problems: Definition of eigenvalue for boundary value problems, and the results. An extra eigenvalue problem is follows:

Exercise 4: Find the eigenvalues and associated eigenfunctions to:

$$\begin{aligned} &y'' + \lambda y = 0, \qquad 0 < x < L; \\ &y'(0) = 0, \quad y(L) = 0. \end{aligned}$$

Solve PDEs: Two steps to solve the problems.
Exercise 5: Consider the initial boundary value problem:

$$\begin{aligned} &u_t = u_{xx}, \qquad 0 < x < L, \quad t > 0; \\ &u_x(0,t) = u(L,t) = 0, \quad t > 0 \\ &u(x,0) = \cos \tfrac{3\pi x}{2L}, \qquad 0 < x < L. \end{aligned}$$

(a). Find two linearly independent non-zero solutions to

$$\begin{aligned} &u_t = u_{xx}, \qquad 0 < x < L, \quad t > 0; \\ &u(0,t) = u(L,t) = 0, \quad t > 0. \end{aligned}$$

(b) Solve above initial boundary value problem.

Formulas will be provided in Final exam: Fourier series formula (including sine and cosine series), Laplace transform formulas.