Review for the course

Final Exam: 12-13-2005, 1:30pm-3:30pm.

Extra office hour: Monday 10:30am-2:30pm

Complex numbers and functions, Chapter 2.

Complex numbers: Definitions, two forms, algebraic operations.

Exercise 1: Find the real and imaginary parts and modules of of following numbers:

(a). $z = (1+i)^{-2i}$; (b). $z = e^{2+4i}$.

Fourier series, Chapter 7

Harmonic functions, periodic functions: Period, amplitude, "orthogonal" of two harmonic functions.

Exercise 2: Computer

$$\int_{-\pi}^{\pi} e^{2xi} \cdot e^{-3xi} dx = ?$$

Fourier series: How to find Fourier coefficients. Parseval's theorem and applications.

Exercise 3:

(a). Find Fourier series for

$$f(x) = \begin{cases} -1, & -l \le x < 0, \\ 1, & 0 \le x < l. \end{cases}$$

(b). Using above Fourier series and Parseval's theorem to computer

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots.$$

First order ODEs

Definition: We start with some terminologies: Differential equations, solutions, general solutions, particular solutions, etc

Exercise 4: Check that$y = ce^{-\alpha x}$ is a general solution to

$$y" - \alpha^2 y = 0.$$

How to solve a differential equation (1st order): 7 types:

Exercise 5: Solve

$$y^2 \frac{dy}{dx} + 2xy^3 = x.$$

High order linear ODEs

Definition: We start with some terminologies: Linear differential equations, superposition, general solutions, homogeneous and nonhomogeneous equations, particular solutions, etc.

Exercise 6: If y_1 and y_2 solve equation

$$y^{''} + e^x y' + y^2 \sin x = 0,$$

is $y_1 + y_2$ also a solution? If $y_1 + y_2$ is still a solution, show that

$$y_1(x) y_2(x) \sin x = 0.$$

How to solve a homogeneous equation of constant coefficients:

$$y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = 0.$$

Three types.

Exercise 7: If the characteristic equation of a ODE is

(a).

$$(r-1)^3(r^2 - 4x + 5) = 0,$$

(b).

$$(r-1)^3(r^2 - 2x + 2)^2 = 0,$$

find the general solutions.

How to solve a nonhomogeneous equation of constant coefficients:

$$y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = f(x).$$

Undetermined coefficient method: Three cases.

Exercise 8: Find the general solution to

$$y" + 4y' + 5 = x + e^{2x}\sin x.$$

solve an equation of constant coefficients by **Laplace transform**:

Exercise 8: Using Laplace transform to find the solution to

$$y" + 4y' + 5 = x + e^{2x}\sin x, \quad y'(0) = 0, \quad y''(0) = 1.$$

Exercise 9: If

$$Y = \frac{p+4}{(p+4)^2+3},$$

find $y(t)$ so that $L\{y(t)\} = Y$.

Eigenvalue problems for boundary value problems: the set-up and its applications to solving PDEs.

Typical example:

Exercise 10: Solve the eigenvalue problems:

$$y'' = \lambda y, \quad y'(0) = y'(\pi) = 0.$$

Here is a fun one:

Exercise 11: Solve the eigenvalue problems:

$$y' = \lambda y, \quad y(0) = 1 \; y(1) = e.$$

Power series to solve ODEs: how to find the recursive relation.

Exercise 12 Using power series to solve

$$y'' = y.$$

(a). Find the recursive relation.

(b). Find the power series solution.

PDEs:

Laplace equation, heat equation and wave equation. Two steps. Step 1: solving homogeneous boundary problem with the help of the solution to eigenvalue problem. Step 2: Solving initial value of whole boundary problem.

Exercise 13 Find three linearly independent solutions to

$$u_x - u_y = 0.$$

Exercise 14 Find three linearly independent solutions to

$$u_x(x,y) - u_y(x,y) = 0, \text{and} \quad u(0,y) = u(\pi,y) = 0.$$

Formulas will be provided: Fourier series formula, Laplace transform formulas.