Review for Midterm 2

Extra office hour: Friday: 12:50-2:30pm

First order ODEs

Definition: We start with some terminologies: Differential equations, solutions, general solutions, particular solutions, etc

Exercise 1: Check $y = e^{-2x}$ solves

$$y'' - 4y = 0.$$

How to solve a differential equation (1st order): 7 types:

Type 1:

$$\frac{dy}{dx} = f(x).$$

We learned it in Calculus course.

Type 2:

$$\frac{dy}{dx} = g(x)h(y).$$

Separable equations! Change it to

$$\frac{dy}{h(y)} = g(x)dx.$$

Caution: $h(y) = 0$ **might be a solution!**

Exercise 2: Solve

$$\frac{dy}{dx} + x = xy^2.$$

Type 3:

$$\frac{dy}{dx} + P(x)y = Q(x).$$

Linear. Find integrating factor.

Exercise 3: Solve

$$\frac{dy}{dx} + xy = x^2.$$

Type 4:

$$\frac{dy}{dx} = F(ax + by + c).$$

Linear substitution: let $v(x) = ax + by + c$.

Exercise 4: Solve

$$\frac{dy}{dx} = (x + y + 2)^2.$$

Type 5:

$$\frac{dy}{dx} = F(\frac{y}{x}).$$

Nonlinear substitution: let $v(x) = y/x$.

Exercise 5: Solve

$$2xy\frac{dy}{dx} = 4x^2 + 3y^2.$$

Type 6:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Bernoulli equations. Change it to

$$\frac{dy}{y^n dx} + P(x)y^{-(n-1)} = Q(x).$$

Then let $v(x) = y^{-(n-1)}$.

Exercise 6: Solve

$$y^2\frac{dy}{dx} + 2xy^3 = 6x.$$

Type 7:

$$M(x, y)dx + N(x, y)dy = 0.$$

Exact equations **?**

Exercise 7: Solve

$$(x^3 + \frac{y}{x})dx + (y^2 + lnx)dy = 0.$$

High order linear ODEs

Definition: We start with some terminologies: Linear differential equations, superposition, general solutions, homogeneous and nonhomogeneous equations, particular solutions, etc.

Exercise 8: If y_1 and y_2 solve equation

$$y^{''} + e^x y' + y \sin x^2 = 0,$$

check $2y_1 + 3y_2$ is also a solution.

How to solve a homogeneous equation of constant coefficients:

$$y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = 0.$$

Three types.

Consider the Characteristic equation.

Type 1: Distinct roots $R_1, ..., R_n$

Type 2: Repeat roots (roots with multiplicity greater than 1)

Exercise 9: Find and check the general solution to

$$y" + 4y' + 4 = 0.$$

Type 3: complex roots.

Exercise 10: If the characteristic equation of a ODE is
(a).

$$(r-1)^3(r^2 - 4x + 5) = 0,$$

(b).

$$(r-1)^3(r^2 - 2x + 2)^2 = 0,$$

find the general solutions.

How to solve a nonhomogeneous equation of constant coefficients:

$$y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = f(x).$$

Undetermined coefficient method: Three cases.

Exercise 11: Find the general solution to

$$y" + 4y' + 5 = x + e^{2x} \sin x.$$