

Hint for exercises in the Review for Midterm 3

Exercise 1:

(a). If $L(f(t)) = F(p)$, show that

$$L(f(t) + 2e^{-3t}) = F(p) + \frac{2}{p+3}.$$

Hint: linearity!

(b). $L(f(t)) = F(p)$, show that

$$L\{e^{-2t}f(t)\} = F(p+2).$$

Hint:

$$\int_0^{\infty} e^{-pt}f(t)e^{-2t}dt = \int_0^{\infty} e^{-pt-2t}f(t)dt = \int_0^{\infty} e^{-(p+2)t}f(t)dt = F(p+2).$$

How to solve an equation of constant coefficients: Second order, or linear systems (for example, Q28 on page 443).

Step 1: Solve $L\{y(t)\} = Y$ algebraically.

Exercise 2:

(a). If $y(t)$ solves

$$y'' + 4y' + 13y = 20e^{-t}, \quad y(0) = 1, \quad y'(0) = 3.$$

Find

$$Y = L\{y(t)\}.$$

Hint: basic algebra

(b). If $y(t)$, $z(t)$ solve

$$y' + 2z = 1, \quad 2y - z' = 2t, \quad y(0) = 0, \quad z(0) = 1.$$

Find

$$Y = L\{y(t)\} \quad \text{and} \quad Z = L\{z(t)\}.$$

Hint: Solve 2x2 system

Step 2. Find $y(t)$ from Y :

Exercise 3:

(a). If

$$Y = \frac{p}{p^2 + 3},$$

find $y(t)$ so that $L\{y(t)\} = Y$.

(b). If

$$Y = \frac{p + 4}{(p + 4)^2 + 3},$$

find $y(t)$ so that $L\{y(t)\} = Y$.

Hint: reviewed last Tuesday. Using formula L29

Eigenvalue problems for boundary value problems: the set-up and its applications to solving PDEs.

Typical example:

Exercise 4: Solve the eigenvalue problems:

$$y'' = \lambda y, \quad y(0) = y(\pi) = 0.$$

Hint: find the solution from your notes.

Power series to solve ODEs: how to find the recursive relation.

Exercise 5 Using power series to solve

$$y' = y.$$

(a). Find the recursive relation.

Hint:

$$\sum_{n=0}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} a_n x^n,$$

thus

$$\sum_{n=1}^{\infty} (n + 1) a_{n+1} x^n = \sum_{n=0}^{\infty} a_n x^n.$$

So

$$a_{n+1} = \frac{a_n}{n + 1}, \text{ for } n = 0, 1, \dots$$

(b). Find the power series solution.

Hint: Now you shall know the answer from part (a).

PDEs: Laplace equation.

Exercise 6 Find three linearly independent solutions to

$$\Delta u(x, y) = 0.$$

Hint: try any such functions:

$$e^{kx} \sin kx, \quad \text{or} \quad e^{kx} \cos kx.$$

Exercise 7 Find three linearly independent solutions to

$$\Delta u(x, y) = 0, \text{ and } u(0) = u(\pi) = 0.$$

Hint: Using Exercise 6 by choosing right k .

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