Review for Midterm 3

Extra office hour: Monday: 12:50-2:30pm

Algebraic method to solve high order linear ODEs

Definition: We start with Laplace transform (section 8.8). The basic algebraic properties about the Laplace transform: linearity.

Exercise 1:

(a). If $L(f(t)) = F(p)$, show that

$$L(f(t) + 2e^{-3t}) = F(p) + \frac{2}{p+3}.$$

(b). $L(f(t)) = F(p)$, show that

$$L\{e^{-2t}f(t)\} = F(p+2).$$

How to solve an equation of constant coefficients: Second order, or linear systems (for example, Q28 on page 443).

Step 1: Solve $L\{y(t)\} = Y$ algebraically.

Exercise 2:

(a). If $y(t)$ solves

$$y'' + 4y' + 13y = 20e^{-t}, \quad y(0) = 1, \quad y'(0) = 3.$$

Find

$$Y = L\{y(t)\}.$$

(b). If $y(t),\ z(t)$ solve

$$y' + 2z = 1,\ 2y - z' = 2t, \quad y(0) = 0, \quad z(0) = 1.$$

Find

$$Y = L\{y(t)\} \ \text{ and } \ Z = L\{z(t)\}.$$

Step 2. Find $y(t)$ from Y:

Exercise 3:

(a). If

$$Y = \frac{p}{p^2+3},$$

find $y(t)$ so that $L\{y(t)\} = Y$.

(b). If

$$Y = \frac{p+4}{(p+4)^2+3},$$

find $y(t)$ so that $L\{y(t)\} = Y$.

Eigenvalue problems for boundary value problems: the set-up and its applications to solving PDEs.

Typical example:

Exercise 4: Solve the eigenvalue problems:

$$y'' = \lambda y, \quad y(0) = y(\pi) = 0.$$

Power series to solve ODEs: how to find the recursive relation.

Exercise 5 Using power series to solve

$$y' = y.$$

(a). Find the recursive relation.

(b). Find the power series solution.

PDEs: Laplace equation.

Exercise 6 Find three linearly independent solutions to

$$\Delta u(x,y) = 0.$$

Exercise 7 Find three linearly independent solutions to

$$\Delta u(x,y) = 0, \text{and} \quad u(0) = u(\pi) = 0.$$

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