Review for Midterm 2

Extra office hour: Wed. 2:30-4:00pm

On Real numbers (Chapter 3).

Topology of the reals, compactness Neighborhood, open set and closed sets, Heine-Borel theorem.

Exercise 1:

(a). If A and B are two open sets in R, prove that $A \cap B$ and $A \cup B$ are both open.

(b)*. If A_i are all open sets in R for $i = 1, 2, \cdots$. Will it be possible that

$$\cup_{i=1}^{\infty} A_i$$

is a compact set? Why?

Sequences (Chapter 4).

limit: Definition, algebraic properties, squeeze theorem.

Exercise 2: (a). If $\{|a_n|\}_{n=1}^{\infty}$ converges to zero, prove that $\{a_n\}_{n=1}^{\infty}$ converges to zero too.

(b). If $\{a_n\}_{n=1}^{\infty}$ converges to L, prove that $\{|a_n|\}_{n=1}^{\infty}$ converges to $|L|$.

(c). Find

$$\lim_{n \to +\infty} (\sqrt{n+1} - \sqrt{n}) = ?$$

Properties about convergence sequence: Convergent sequence, Monotonic bounded sequence, Cauchy sequence, subsequence.

Exercise 3. Let a_n be a sequence in R. If $\lim_{n\to\infty} a_{2n} = \lim_{n\to\infty} a_{2n+1} = L$, prove that $\lim_{n\to\infty} a_n = L$.

Exercise 4. For a given sequence a_n, if $\lim_{n\to\infty}(a_{n+1} - a_n) = 0$, can you conclude that $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence? Why?

limit for function (Chapter 5)

Limit, continuity: Limit for function, continuous function, algebraic properties.

Exercise 5: State and prove the squeeze theorem for the limit of function $f(x)$ at a point $x = c$.

Exercise 6: Prove: if $f(x)$ is continuous at $x = c$, then $|f(x)|$ is continuous at $x = c$.

WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!

Comments and question to: mzhu@math.ou.edu