

Review for final

Extra office hour: Monday, Tuesday 2:00–4:30pm.
Exam covers 1.1-6.4

Previous review is good for (Chapter 1–Chapter 4).

Continuity for functions (Chapter 5).

Basic algebraic properties: Algebraic operations and composition functions.

Exercise 1. (a) If $f(x)$ is continuous on $I = (0, 1)$ and $f(x)$ is positive at $c \in (0, 1)$. Prove that there is a $\delta > 0$ such that $f(x) > 0$ in $V_\delta(c)$.

(b) Find two functions defined in R such that both of them are discontinuous everywhere, but $f + g$ is continuous everywhere.

(c). Find two functions defined in R such that both of them are discontinuous everywhere, but $g \circ f$ is continuous everywhere.

Continuous functions on closed interval: extremum, intermediate value theorem, uniformly continuous.

Exercise 2:

(a). Give an example (and prove your statement) that a continuous function $f(x)$ in an open interval $(1, 2)$ may not be uniformly continuous.

(b). Let $f(x)$ be a function defined in $[a, b]$. If $f(x)$ is continuous in (a, b) and $f(x)$ is differentiable at a and b , prove that $f(x)$ is uniformly continuous in $[a, b]$.

(c). If $f(x)$ is continuous on $I = (0, 1)$ and $f(x)$ is positive at on I . Can you say that $f(x) > a$ on I for some positive number a ? If the interval I is a closed one, say, $I = [0, 1]$, can you show that there is a $a > 0$ such that $f(x) > a$ for $x \in I$?

Exercise 3: Prove Theorem 5.6.5 on page 152.

Derivatives for functions (Chapter 6).

Basic algebraic properties and Chain rule:

Exercise 4: (a). If $f(x)$ is differentiable at $x = c \in I$, where I is an interval, prove that $f(x)^2$ is differentiable at $x = c \in I$. Show that $f(x)^{1/3}$ might not be differentiable at $x = c \in I$.

(b). The proof of Chain rule.

Mean value theorem: Various forms

Exercise 5: Assume that $f(x)$ and $g(x)$ are two continuous function in $[a, b]$. If $f(x)$ and $g(x)$ are differentiable in (a, b) , and $g'(x) \neq 0$ in (a, b) , show that there is a $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.$$

Further applications and generalizations of Mean value theorem: L'Hospital's Rule and its application; Taylor's theorem and its applications.

Exercise 6: (a). Find the value

$$\lim_{x \rightarrow 0} \frac{\sin x}{x + 1} =$$

(b). What is wrong when you apply L'Hospital's rule in the following solution?

$$\lim_{x \rightarrow 0} \frac{\sin x}{x + 1} = \lim_{x \rightarrow 0} \frac{(\sin x)'}{(x + 1)'} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1.$$

Exercise 7: (a). Let $f(x)$ be an even continuous function on $(-\infty, +\infty)$ and $\lim_{x \rightarrow \infty} f(x) = L$. Prove that $f(x)$ is uniformly continuous.

(b). Let $f(x)$ be an even differentiable function on $(-\infty, +\infty)$. If $\lim_{x \rightarrow \infty} (f(x) + f'(x)) = L$, show that $\lim_{x \rightarrow \infty} f'(x) = 0$.

Exercise 8: (a). From the Mean value theorem prove that there is a $\theta \in (0, x)$ for every positive x , such that

$$\ln(1 + x) = \frac{x}{1 + \theta x}.$$

*(b). Prove that

$$\lim_{x \rightarrow +0} \theta = \frac{1}{2}.$$

This one is a little bit hard, but interesting.

WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!

Comments and question to: mzhu@math.ou.edu

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