Be sure that this examination has 1 page.

The University of Oklahoma
Final Examinations - May 08, 2005

Intro. Analysis
Section 01

Dr. M. Zhu

Open book examination | Time: minutes

Choose three questions from the following four questions

Marks

[30] **1.** Let $f(x) \in C^2[0,1]$.

a) Write down the Taylor series up to the second derivative of $f(x)$ at point $x = \dfrac{1}{1+\lambda}$, where $\lambda > 0$.

b) If $f"(x) > 0$ for $x \in [0,1]$, prove that for any $\lambda > 0$,

$$\int_0^1 f(x^\lambda)dx \geq f(\frac{1}{1+\lambda}).$$

[30] **2.** Let $f(x) : E^2 \to R$ be a function of two variables. Prove that f is a function of homogeneous degree n if and only if

$$x_1 \partial f/\partial x_1 + x_2 \partial f/\partial x_2 = nf(x).$$

Please state and prove the analog result for a function of k variables.

[30] **3.** Let $f(x,y) : E^2 \to R$ be a function of two variables.

a) Give an example that $f(x,y)$ is continuous at a point (x_0, y_0) with respect to each variable, but it is not continuous at this point as a function of two variables.

b) If $f(x, y_0)$ is a continuous function at $x = x_0$ and there is a constant L such that

$$|f(x,y_1) - f(x,y_2)| \leq L|y_1 - y_2|, \quad \forall x \in R.$$

Prove that $f(x,y)$ is continuous at (x_0, y_0).

[30] **4.** Let $f(t) \in C[0,1]$. Prove for any given λ such that $|\lambda| < 1$, there is a unique function $u(t) \in C[0,1]$ satisfying

$$u(t) - \lambda \int_0^1 e^{t-s} u(s)ds = f(t).$$

[120] **Total Marks**

The End