Review for Midterm

Extra office hour: Wednesday 9:30-11:30pm

On Introduction (Chapter 1).

Partial Differential Equations: Notations: order, linearity, homogeneity; Solutions: Classic solutions, weak solutions.

Exercise 1: a). Show that $u(x,t) = e^{x-at}$ is a solution to $a^2 u_{xx} = u_{tt}$.

b). Show that $u(x,t) = f(x-at)$ is a solution to $a^2 u_{xx} = u_{tt}$, where $f(z)$ is a function with continuous second order derivative.

On three equations (Chapter 2).

Transport Equations: Homogeneous initial problem. Nonhomogeneous problem.

Exercise 2: Using Duhamel principle to derive the formula for nonhomogeneous problem:

$$\begin{cases} u_t + b \cdot Du &= f \quad in \quad R^n \times (0,\infty) \\ u &= 0 \quad on \quad R^n \times (t=0). \end{cases}$$

Laplace's equation: Fundamental solution, mean value theorem, maximum principle, Harnack inequality, regularity.

Exercise 3: Write down one positive harmonic function in (1): Whole R^n; (2): Upper half plane R^n_+; (3): Punctured ball $B_1(0) \setminus \{0\}$.

Exercise 4: Prove that if $u(x)$ is a harmonic function in R^n, then $\frac{1}{|x|^{n-2}} u(\frac{x}{|x|^2})$ is a harmonic function in $R^n \setminus \{0\}$.

Heat equation: Fundamental solution, mean value theorem, maximum principle, energy identities.

Exercise 5: Write down the backwards uniqueness theorem and proof by yourself.

Please also do your own homework.

WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!

Comments and question to: mzhu@math.ou.edu