

MATH 1823 Honors Calculus I Irrational Numbers

Due in class on Friday, Sept 8, 2000

Classical belief. The Pythagorean school believed that you could obtain any number (measurement) you like by taking the ratio of two whole numbers (integers). That is, they believed that all numbers were rational. On one level, this might seem to be a reasonable belief. You see you cant make all possible measurements using whole numbers (integers), since the gap between successive whole numbers is 1. We can introduce the fractions $n/2$ to make these gaps all be $1/2$. In general, if we have two fractions, no matter how close together, then their average will be a third fraction whose distance from the first two will be exactly half the distance that they were apart.

- Show that the average of p/q and r/s is another rational number.
- Draw a picture of these three numbers, starting from the assumption that $p/q < r/s$.

Therefore, since the gap length can be made to go to zero, there must be no gaps at all between the rational numbers. That is, every measurement (number) we want is rational.

Some problem numbers. Here you see that there are some numbers which are not rational.

1. Show that $\sqrt{2}$ (the length of the diagonal of a square of side length 1) is not a rational number.
2. Show that $\frac{5\sqrt{2}}{24}$ is not rational. [Hint: Suppose that there are whole numbers p and q such that $p/q = \frac{5\sqrt{2}}{24}$. Do you see any contradiction?] Prove that there are infinitely many irrational numbers.
3. How might you prove that $\sqrt[3]{2}$ is not a rational number?
4. How might you prove that $\sqrt{3}$ is not a rational number?
5. Do a net-search to find answers to the following questions. Say which search engines you used, and which keywords you used!
 - (a) Is π rational?
 - (b) Is e rational?
 - (c) Are there more rational numbers than irrational numbers?

Conclusions. Now, something must be flawed with the Pythagorean reasoning at the top of the page. Where is the gap? Can you reconcile the two sections of this page [one: gaps between rationals can be made to go to zero, and two: there exist irrational numbers]?