Q1]... [10 points] Complete the truth table for the proposition below. The extra columns are there for you to use to compute truth values of intermediate expressions (if you wish).

P	Q	R	$\neg Q$	$R \to \neg Q$	Negat	$P \wedge \neg R$	$\neg(R \to \neg Q) \vee (P \wedge \neg R)$
T	T	T	F	F	T	F	T
T	T	F	F	T	F	T	T
T	F	T	T	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	F	T
F	T	F	F	T	F	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	F	F	F

Mid I – Solutions

Q2]. . . [10 points] Find a **disjunctive normal form** expression (involving $\wedge$, $\vee$, $\neg$, and P, Q, R) which has the following truth table. Show the steps of your work.

P	Q	R	
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

$(P\wedge Q\wedge R)$ has T as 1st output, F's elsewhere

$(P\wedge Q\wedge \neg R)$ has T -- 2nd — — — — —

$(P\wedge \neg Q\wedge R)$ has T — — — 3rd — — — —

$(\neg P\wedge \neg Q\wedge R)$ has T as 7th output, F's elsewhere

$\Rightarrow$ $(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(P\wedge\neg Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)$ works $\boxed{\text{DNF}}$

Find a **conjunctive normal form** expression (involving $\wedge$, $\vee$, $\neg$, and P, Q, R) which has the same truth table above. Show the steps of your work.

$(\neg P\vee Q\vee R)$ has F as 4th output, T's elsewhere

$(P\vee\neg Q\vee\neg R)$ has — — 5th — — — —

$(P\vee\neg Q\vee R)$ — — — — 6th — — — —

$(P\vee Q\vee R)$ — — — — 8th — — — —

$\Rightarrow$ $(\neg P\vee Q\vee R)\wedge(P\vee\neg Q\vee\neg R)\wedge(P\vee\neg Q\vee R)\wedge(P\vee Q\vee R)$

works $\boxed{\text{CNF}}$

Q3]... [10 points] Let $P(x,y)$ be the statement "x and y satisfy the equation $2x+y=4$". Determine which of the following are true; the universe for x and y is the set of real numbers. Justify your answers.

1. $\forall x \exists y P(x,y)$ True — Given a number x, then $y = 4 - 2x$ works!

2. $\forall x \forall y P(x,y)$ False! — eg $x=7$, $y=7$ $\quad 2(7)+(7) \neq 4$

3. $\exists x \exists y P(x,y)$ True: — eg $x=1, y=2$ works $\quad 2(1)+(2)=4$

4. $\exists x \forall y P(x,y)$ False: Once we select x, only one value of y will work; namely, $y = 4-2x$

Let the universe of x be all the people in the world, let $F(x)$ denote "x is friendly", let $T(x)$ denote "x is tall", and $A(x)$ denote "x is angry". Translate the following statements into predicate statements with suitable quantifiers.

1. All tall people are friendly. $\quad \forall x (T(x) \rightarrow F(x))$

2. Some tall people are friendly. $\quad \exists x (T(x) \wedge F(x))$

3. No friendly people are angry. $\quad \forall x (F(x) \rightarrow \neg A(x))$ which is equivalent to $\neg(\exists x (F(x) \wedge A(x)))$.

4. There is precisely one tall, angry person.

$$\exists x \left[(T(x) \wedge A(x)) \wedge \forall y \left((y \neq x) \rightarrow \neg (T(y) \wedge A(y)) \right) \right]$$

Q4]... **[10 points]** Give a **direct proof** of the following. *If m and n are odd integers, then their product is also odd.*

Pf. m odd $\Rightarrow$ $m = 2k+1$ for some integer k.

n odd $\Rightarrow$ $n = 2l+1$ for some integer l.

Then $mn = (2k+1)(2l+1)$

$= (2k)(2l) + (2k)(1) + (1)(2l) + (1)(1)$

$= 2(2kl + k + l) + 1$

which is of the form 2 times an integer plus 1.

Therefore mn is odd. $\boxed{}$

Prove the following **by contradiction**. *If n is an integer and $3n^2 + 8$ is even, then n is also even.*

Pf by contradiction:

Assume conclusion is false.

That is, n is not even

$\Rightarrow$ n is odd

$\Rightarrow$ $n^2 = (n)(n)$ is odd ... by 1st part (above).

$\Rightarrow$ $3n^2 = (3)(n^2)$ is odd ... again by 1st part (above)

~~$\Rightarrow$ $3n^2 + 8$ is odd (why? if $3n^2 +$~~

$\Rightarrow$ $3n^2 = 2j + 1$ for some integer j

$\Rightarrow$ $3n^2 + 8 = (2j+1) + 8 = 2(j+4) + 1$

$\Rightarrow$ $3n^2 + 8$ is odd. But this contradicts the hypothesis that $3n^2+8$ is even. $\Rightarrow$ done! $\boxed{}$

Q5]. . . [10 points] Suppose $A = \{a, b, c\}$. Say whether the following are true or false.

1. $\{a\} \in A$. No

2. $\{a\} \in \mathcal{P}(A)$. yes

3. $b \in A$ yes

4. $\emptyset \in \mathcal{P}(A)$. yes

5. $\{\emptyset\} \subset \mathcal{P}(A)$. yes

6. $\{a, b\} \in \mathcal{P}(A)$. yes

7. $(a, c) \in A \times A$. yes

8. $|A \times A| = 2^3$. NO - - - $|A \times A| = 3^2$

9. $|\mathcal{P}(A)| = 3^2$. NO - - - - $|P(A)| = 2^3$

10. $|\{(x, y) \in A \times A \mid x \neq y\}| = 6$. yes

$|\{(a,b), (a,c), (b,c), (c,a), (c,b), (b,a)\}|$