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Homework #3

1. $f(x) = x^4$.

Find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

$$f(x+h) = (x+h)^4$$
$$= (x+h)^3 (x+h)$$
$$= (x^3 + 3x^2h + 3xh^2 + h^3)(x+h) \quad \left[\text{using } (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\right]$$
$$= x^4 + 3x^3h + 3x^2h^2 + xh^3 + x^3h + 3x^2h^2 + 3xh^3 + h^4$$
$$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$\therefore \frac{f(x+h) - f(x)}{h} = \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - \cancel{x^4}}{h}$$
$$= \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^3)}{\cancel{h}}$$
$$= 4x^3 + 6x^2h + 4xh^2 + h^3 \quad \#$$

\# 24. $\lim_{x \to 5^-} \frac{6}{x-5}$.

Since $\frac{6}{x-5} < 0$ for $x < 5$, and use defn of limit

$\lim_{x \to 5^-} \frac{6}{x-5} = -\infty$

\# 18. $\lim_{x \to 1} \frac{x^3-1}{x^2-1} = \lim_{x \to 1} \frac{x^3-1^3}{x^2-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}}$

$= \boxed{\frac{3}{2}}$

\# 22 $\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h} \times \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$

$= \lim_{h \to 0} \frac{\cancel{1}+h-\cancel{1}}{h(\sqrt{1+h}+1)}$

$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)}$

$= \boxed{\frac{1}{2}}$

#28. $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{(3+h)} - \frac{1}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3} - \cancel{3} - h}{h \cdot 3\,(3+h)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\, 3\,(3+h)}$$

$$= -\frac{1}{3 \cdot 3} = \boxed{-\frac{1}{9}}$$