Solutions (Homework-8)

(3) $y = (1-x^2)^{10}$

$y = u^{10}, \quad u = 1-x^2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 10u^9(-2x) = 10(1-x^2)^9(-2x)$$

$$\boxed{\frac{dy}{dx} = -20x(1-x^2)^9}$$

(4) $y = \tan(\sin x)$

$y = \tan u, \quad u = \sin x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \sec^2 u(\cos x) = \sec^2(\sin x)\cos x$$

$$\boxed{\frac{dy}{dx} = \sec^2(\sin x)\cos x}$$

(6) $y = \sin\sqrt{x}$

$y = \sin u, \quad u = \sqrt{x}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u\left(\frac{1}{2\sqrt{x}}\right) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos\sqrt{x}}{2\sqrt{x}}}$$

(10) $y=f(x)=(1+x^4)^{2/3}$

$y=(u)^{2/3} \quad u=1+x^4$

$$\frac{dy}{dx}=f'(x)=\frac{dy}{du}\cdot\frac{du}{dx}=\tfrac{2}{3}(u)^{1/3}(4x^3)$$

$$=\frac{8}{3}(1+x^4)^{1/3}x^3$$

$$\boxed{f'(x)=\frac{8}{3}x^3(1+x^4)^{1/3}}$$

(21) $y=x^3\cos nx$

By product rule,

$$\frac{dy}{dx}=\frac{d}{dx}(x^3)\cos nx+x^3\frac{d}{dx}(\cos nx)$$

$$=3x^2\cos nx-nx^3\sin nx$$

$$\boxed{\frac{dy}{dx}=3x^2\cos nx-nx^3\sin x}$$

(22) $y=x\sin\sqrt{x}$

$$\frac{dy}{dx}=\frac{d}{dx}(x)\sin\sqrt{x}+x\frac{d}{dx}(\sin\sqrt{x})$$

$$\boxed{\frac{dy}{dx}=\sin\sqrt{x}+\frac{x\cos\sqrt{x}}{2\sqrt{x}}}$$

(24) $y = f(x) = \dfrac{x}{\sqrt{7-3x}}$

$$\frac{dy}{dx} = f'(x) = \frac{\sqrt{7-3x}\,\frac{d}{dx}(x) - x\,\frac{d}{dx}\left(\sqrt{7-3x}\right)}{7-3x}$$

$$= \frac{\sqrt{7-3x} - \dfrac{x}{2\sqrt{7-3x}}(-3)}{7-3x}$$

$$= \frac{\left(\dfrac{2(7-3x)+3x}{2\sqrt{7-3x}}\right)}{7-3x}$$

$$= \frac{14-3x}{2(7-3x)^{3/2}}$$

$$\boxed{f'(x) = \frac{14-3x}{2(7-3x)^{3/2}}}$$

(32) $y = \tan^2(3\theta)$

$y = \tan^2(u)\,, \quad u = 3\theta$

$y = v^2\,, \quad v = \tan u$

$$\frac{dy}{d\theta} = \frac{dy}{dv}\cdot\frac{dv}{du}\cdot\frac{du}{d\theta} = 2v\cdot\sec^2 u\cdot 3 = 6v\sec^2 u$$

$$\boxed{\frac{dy}{dx} = 6\tan(3\theta)\sec^2(3\theta)}$$

(39) $y = \sqrt{x + \sqrt{x}}$

$y = \sqrt{u}, \quad u = x + \sqrt{x}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}}\right)$$

(42) $y = \sqrt{\cos(\sin^2 x)}$

$y = \sqrt{\cos u^2}, \quad u = \sin x$

$y = \sqrt{\cos v}, \quad v = u^2$

$y = \sqrt{w}, \quad w = \cos v$

$$\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{w}} - \sin v \cdot 2u \cdot \cos x$$

$$= -\frac{u \sin v \cos x}{\sqrt{w}}$$

$$\frac{dy}{dx} = -\frac{\sin x \left(\sin(\sin^2 x)\right) \cos x}{\sqrt{\cos(\sin^2 x)}}$$

$$\boxed{\frac{dy}{dx} = -\frac{\sin(\sin^2(x)) \sin x \cos x}{\sqrt{\cos(\sin^2 x)}}}$$

(66) $s = A(\cos(\omega t + \delta))$

(a) Velocity at time $(t) = \frac{ds}{dt} = -A\omega \sin(\omega t + \delta)$

(b) Velocity $= 0$

$\Rightarrow -A\omega \sin(\omega t + \delta) = 0$

$\Rightarrow \sin(\omega t + \delta) = 0$

$\Rightarrow \delta + \omega t = n\pi \quad (n \in \mathbb{Z})$

$\Rightarrow t = \frac{n\pi - \delta}{\omega}$

Velocity $= 0$ when $t = \frac{n\pi - \delta}{\omega}$