

Friday 12/15/2006

Final Examination

10:30am-12:30pm

Name: Student ID: **Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	10	
Q2	10	
Q3	15	
Q4	10	
Q5	10	
Q6	10	
Q7	15	
Q8	10	
Q9	10	
TOTAL	100	

Q1... [10 points] Write down the limit definition of the derivative, $f'(x)$, of the function $f(x)$.

Use the limit definition to compute the derivative of the function $f(x) = \sqrt{x}$. Show all the steps of your work. [Do not use power rule!]

Q2]... [10 points] Show that the two curves $y = x^2$ and $xy^2 = 1$ intersect orthogonally. That is, show that their tangent lines at the points of intersection are perpendicular.

Q3]... [15 points] Compute the 30th derivative $f^{(30)}$ of the following functions f .

$$f(x) = x^{30} + 30x^{15} + 15$$

$$f(x) = \sin(7x + 1)$$

$$f(x) = \frac{1}{3x - 1}$$

Q4]... [10 points] Compute the derivatives of the following functions.

$$f(x) = \sqrt{2 + \sin(x^2 + 4)}$$

$$f(x) = \frac{\cos(x)}{\tan(x) + 1}$$

Q5]... [10 points] How fast must you let line out if the kite you are flying is 30 ft above you, 40 ft horizontally away from you, and is moving horizontally away from you at a rate of 10 ft/min?

Q6]. . . [10 points] Find the largest area of a rectangle which can be inscribed inside a semi-circle of radius R , if one side of the rectangle lies along the diameter of the circle.

Q7]... [15 points] Consider the function

$$f(x) = 5x^4 - 4x^5$$

Answer the following questions about $f(x)$ and **draw the graph** of $f(x)$.

1. Determine the x - and y -intercepts of f .
2. Does the graph of $f(x)$ have any vertical asymptotes? Does the graph of $f(x)$ have any horizontal asymptotes?
3. Compute $f'(x)$.
4. Find the critical points, intervals where f is increasing/decreasing and local max/min for f .

5. Compute $f''(x)$.

6. Find intervals where f is concave up and where f is concave down, and inflection points.

Sketch the graph of $f(x)$ indicating the information above.

Q8]... [10 points] Write down the formula for the linearization of the differentiable function $f(x) = \cos(x)$ at the point $x = \pi/4$.

Use the linearization above to estimate $\cos(46^\circ)$.

Q9]. . . [10 points] Write down the Newton's method formula for approximating the root of a differentiable function $f(x)$.

Use Newton's method to write down a formula for approximating the x -coordinate of the point of intersection of the line $y = 2x$ and the function $y = \cos(x)$. If your first guess is $x_1 = \pi/2$, then what value does your formula give for x_2 ?

Bonus Question. [Only attempt this if you have already attempted all other questions, and if you have time left] The functions f and g are differentiable. Also $f'(4) = 3$, $f'(5) = 12$, $f'(7) = 2$, $g'(5) = 7$, and $g(5) = 4$. Determine the following.

$$(f \circ g)'(5)$$

$$\lim_{x \rightarrow 4} f(x) - f(4)$$

$$\lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x^2 - 16}$$

$$\lim_{h \rightarrow 0} \frac{f(4+h) - f(4-h)}{h}$$