Monday 09/23/2013 Midterm I 1:30pm–2:45pm

Name: Student ID:

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	10	
Q2	15	
Q3	15	
Q4	15	
Q5	15	
Q6	15	
Q7	15	
TOTAL	100	

Q1]. . . [10 points] Suppose f is a function which is differentiable at the input a. Write down the limit definition of the derivative $f'(a)$.

Write down two interpretations of the derivative $f'(a)$.

Q2]. . . [15 points] For each of the two functions below, say if the function is continuous at 0, and also say if the function is differentiable at 0.

$$f(x) \;=\; \begin{cases} x\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$g(x) \;=\; \begin{cases} 2x+1 & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$$

Q3]. . . [15 points] Compute the following limit. Show all the steps of your work.

$$\lim_{x \to 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}$$

The limit above is the derivative $f'(a)$ of some function $f(x)$ at some input value a. Write down the function $f(x)$ and the input a.

Q4]. . . [15 points] Compute the following limit. Show all the steps of your work.

$$\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$$

The limit above is the derivative $g'(b)$ of some function $g(x)$ at some input value b. Write down the function $g(x)$ and the input b.

Q5]. . . [15 points] Using the limit definition of the derivative, compute the derivative $f'(x)$ of the function $f(x) = x^2 + 3$.

Find points $(a, f(a))$ on the graph of the function $f(x) = x^2 + 3$ above at which the tangent line to the graph also passes through the point $(1, 0)$.

Q6]. . . [15 points] State the intermediate value theorem.

Using the intermediate value theorem, or otherwise, show that there is an input between 0 and π at which the functions $y = \sin(x)$ and $y = x - 1$ have the same output values.

Q7]. . . [15 points] Write down the value of the following limits:

$$\lim_{x\to 0} \frac{\sin(x)}{x} \; = \qquad\qquad \text{and} \qquad \lim_{x\to 0} \frac{\cos(x)-1}{x} \; =$$

Write down the angle addition formula for the sine function:

$$\sin(A+B) \; =$$

Use the limit definition of the derivative to compute the derivative $f'(x)$ of the function $f(x) = \sin(x)$. Show all the steps in your work.

Bonus Question]... Suppose that $f(x)$ is differentiable at a.
Show that

$$\lim_{h\to 0} \frac{f(a) - f(a-h)}{h}$$

exists and is equal to $f'(a)$.

Show that

$$\lim_{h\to 0} \frac{f(a+h) - f(a-h)}{2h}$$

exists and is equal to $f'(a)$.

Suppose that g(x) is a function and that the limit

$$\lim_{h\to 0} \frac{g(a+h) - g(a-h)}{2h}$$

exists, and is some finite number L. Does his imply that $g(x)$ is differentiable at a, and that $g'(a) = L$?