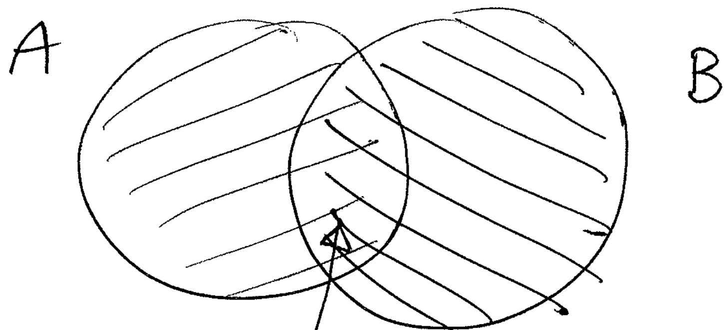
Inclusion - Exclusion for 2

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|} \quad \text{— I}$$



(Elements of $A \cap B$ counted twice when you consider $|A| + |B|$)!

Inclusion - Exclusion for 3

$$\boxed{\begin{aligned} |A_1 \cup A_2 \cup A_3| = {} & |A_1| + |A_2| + |A_3| \\ & - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| \\ & + |A_1 \cap A_2 \cap A_3| \end{aligned}} \quad \text{— II}$$

Proof of II (using I) ....

$$|A_1 \cup A_2 \cup A_3| = |A_1 \cup (A_2 \cup A_3)|$$

Use I with
$A = A_1$
$B = A_2 \cup A_3$

$$= |A_1| + |A_2 \cup A_3| - |A_1 \cap (A_2 \cup A_3)|$$

Use I again with $A = A_2$
$B = A_3$

$$= |A_1| + \overbrace{|A_2| + |A_3| - |A_2 \cap A_3|} - |A_1 \cap (A_2 \cup A_3)|$$

distrib law of $\cap$ over $\cup$.

$$= |A_1| + |A_2| + |A_3| - |A_2 \cap A_3| - |(A_1 \cap A_2) \cup (A_1 \cap A_3)|$$

Use I again with
$A = A_1 \cap A_2$
$B = A_1 \cap A_3$

$$= |A_1| + |A_2| + |A_3| - |A_2 \cap A_3| - \left(|A_1 \cap A_2| + |A_1 \cap A_3| - |A_1 \cap A_2 \cap A_1 \cap A_3|\right)$$

$$= |A_1| + |A_2| + |A_3| - |A_2 \cap A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

done!