

HW #27 (MATH-2924)
SOLUTIONS

Section 12.4

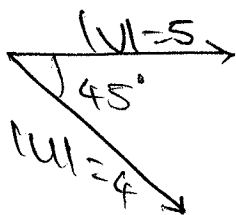
#2

$$a = \langle 1, 1, -1 \rangle, b = \langle 2, 4, 6 \rangle$$

$$\begin{aligned} a \times b &= \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & 4 & 6 \end{vmatrix} = i(6+4) - j(6+2) + k(4-2) \\ &= 10i - 8j + 2k \\ &= \langle 10, -8, 2 \rangle \end{aligned}$$

$$\left. \begin{array}{l} a \cdot (a \times b) = 10 - 8 - 2 = 0 \\ b \cdot (a \times b) = 20 - 32 + 12 = 0 \end{array} \right\} \Rightarrow a \times b \text{ is Orthogonal to } a \text{ \& b.}$$

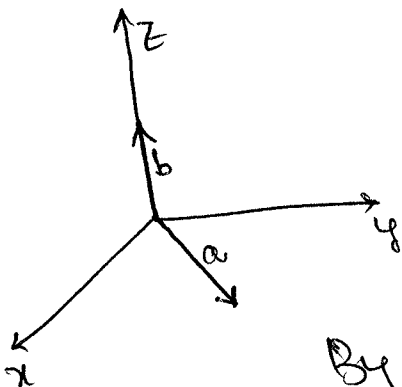
#14



$$\begin{aligned} |u \times v| &= |u| |v| \sin \theta \\ &= 5 \cdot 4 \cdot \sin(45^\circ) = 20 \cdot \frac{1}{\sqrt{2}} \\ &= 10\sqrt{2} \end{aligned}$$

$u \times v$ is directed out of page.

#16

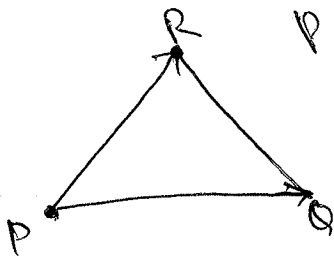


$$|a| = 3, |b| = 2, \text{ angle b/w } a \text{ \& } b \text{ is } 90^\circ$$

$$|a \times b| = |a| |b| \sin \theta = 3 \cdot 2 = 6$$

By Right hand Rule, x coordinate is positive, y coordinate is Negative & z coordinate = 0

#29



$$P = (1, 0, 1), Q = (-2, 1, 3), R = (4, 2, 5)$$

$$\vec{PQ} = (3, 1, 2), \vec{PR} = (3, 2, 4)$$

$\vec{PQ} \times \vec{PR}$ is perpendicular to the plane. (or $\vec{PR} \times \vec{PQ}$)

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = i(4-4) - j(-12-6) + k(-6-3)$$

$$= 18j - 9k = (0, 18, -9)$$

[or (0, -18, 9)]

Area of triangle = $\frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} \sqrt{(18)^2 + (9)^2} = \frac{9}{2} \sqrt{5}$.

#30

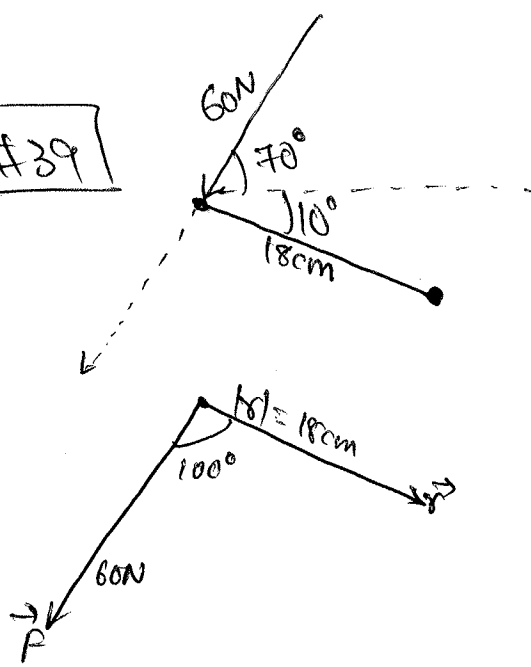
Volume of parallelepiped = $|a \cdot (b \times c)|$

$$\left. \begin{aligned} a &= \langle 1, 2, 3 \rangle \\ b &= \langle -1, 1, 2 \rangle \\ c &= \langle 2, 1, 4 \rangle \end{aligned} \right\}$$

$$V = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} = 1(4-2) - 2(-4-4) + 3(-1-2)$$

$$= 2 + 16 - 9 = 9$$

#39



$$\vec{\tau} = \text{Torque} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}| |\vec{F}| \sin \theta$$

$$= (0.18) (60) \sin 100^\circ$$

$$\approx 10.6 \text{ Joules}$$

converted to meters.