Some results about series.

1. **Geometric Series.**
   $\sum_{n=1}^{\infty} ar^{n-1}$ converges when $|r| < 1$; it converges to the sum $\frac{a}{1-r}$ when $|r| < 1$.

2. **Test for Divergence.**
   If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

3. **Integral Test.**
   For $f(x)$ continuous on $[1, \infty)$, positive and decreasing to 0, the series $\sum_{n=1}^{\infty} f(n)$ converges if and only if the improper integral $\int_1^{\infty} f(x)dx$ converges.

4. **Comparison Tests.**
   Direct comparison test: compares series of positive terms, term-by-term.

   Limit comparison test: compares series of positive terms $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ when $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ a finite limit not equal to 0.

5. **Root Test.**
   Let $\lim_{n\to\infty} |a_n|^{1/n} = L$. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and if $L > 1$ then it is divergent.

6. **Ratio Test.**
   Let $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$. If $L < 1$ then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, and if $L > 1$ then it is divergent.

7. **Alternating Series Test.**
   If a_n are positive, decreasing to 0, then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ is convergent. Moreover, the nth partial sum is within a_{n+1} of the sum of the whole series.

8. **Power series.**
   Ratio test is useful for computing the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$.

9. **Taylor and Maclaurin Series.**
   Taylor series for $f(x)$ centered about a is given by

   $$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

   Maclaurin series for $f(x)$ is the Taylor series for $f(x)$ centered about 0.

10. **Remainder Estimate.**
    Taylor's inequality states that if $|f^{(n+1)}(x)| \leq M$ on the interval $[a-d, a+d]$, then

    $$|f(x) - T_n(x)| \;\leq\; \frac{M|x-a|^{(n+1)}}{(n+1)!}$$

    on the interval $[a-d, a+d]$. Here $T_n(x)$ is the *degree n Taylor polynomial approximation* to $f(x)$.