PRINT NAME: SOLUTIONS

Honors Calculus II [2423-001] Quiz I

Q1]...[10 points] Write down the expressions (formulas) for

$$\sum_{i=1}^{n} i \qquad \text{and for} \qquad \sum_{i=1}^{n} i^2$$

Solutions:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

and

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Compute the definite integral below using limits of Riemann sums.

$$\int_1^2 x^2 - x\, dx$$

Solution: Using n subintervals of equal width, we see that the widths are $\triangle x_i = (2-1)/n = 1/n$, and that the right-hand endpoints of the intervals are $x_i = 1 + i/n$ for $1 \leq i \leq n$.

Thus the Riemann sums (using right-hand endpoints) become

$$\begin{aligned}
\sum_{i=1}^{n} \left((1+i/n)^2 - (1+i/n)\right) \triangle x_i &= \sum_{i=1}^{n} \left(1 + 2i/n + i^2/n^2 - 1 - i/n\right)(1/n) \\
&= (1/n^2)\sum_{i=1}^{n}(i + i^2/n) \\
&= (1/n^2)[n(n+1)/2 + n(n+1)(2n+1)/6n] \\
&= \frac{3n^2 + 3n + 2n^2 + 3n + 1}{6n^2} \\
&= \frac{5n^2 + 6n + 1}{6n^2}
\end{aligned}$$

which tends to $5/6$ as $n \to \infty$.