Homework 3

Section 5.5

2) $\int x(4+x^2)^{10}\,dx = ?$

Let $u = 4+x^2$ then $du = 2x\,dx$ hence

$$\int x(4+x^2)^{10}\,dx = \int u^{10}\cdot\frac{du}{2} = \frac{u^{11}}{22}+c$$
$$= \frac{(4+x^2)^{11}}{22}+c$$

6) $\int \cos^4\theta \sin\theta\, d\theta = ?$

Let $u = \cos\theta$ then $du = -\sin\theta\, d\theta$ hence

$$\int \cos^4\theta \sin\theta\, d\theta = \int u^4\cdot(-du) = -\frac{u^5}{5}+c$$
$$= -\frac{(\cos\theta)^5}{5}+c$$

12) $\int \frac{x}{(x^2+1)^2}\,dx = ?$

Let $u = x^2+1$ then $du = 2x\,dx$ hence

$$\int \frac{x}{(x^2+1)^2}\,dx = \int \frac{1}{u^2}\,\frac{du}{2} = \frac{-1}{2u}+c = \frac{-1}{2(x^2+1)}+c$$

22) $\int (1+\tan\theta)^5 . \sec^2\theta \, d\theta = ?$

Let $u = 1+\tan\theta$ then $du = \sec^2\theta \, d\theta$ hence

$$\int (1+\tan\theta)^5 \sec^2\theta \, d\theta = \int u^5 du = \frac{u^6}{6} + c$$

$$= \frac{(1+\tan\theta)^6}{6} + c$$

$\int \frac{x^2}{\sqrt{1-x}} dx = ?$

32) Let $u = 1-x$ then $du = -dx$ hence

$$\int \frac{x^2}{\sqrt{1-x}} dx = \int \frac{(1-u)^2}{\sqrt{u}} . (-du) = -\int \frac{1-2u+u^2}{\sqrt{u}} du$$

$$= -\int \frac{1}{\sqrt{u}} du + \int \frac{2u \, du}{\sqrt{u}} - \int \frac{u^2}{\sqrt{u}} du$$

$$= -2\sqrt{u} + \frac{4}{3} u^{3/2} - \frac{2}{5} u^{5/2} + c$$

$$= -2\sqrt{1-x} + \frac{4}{3}(1-x)^{3/2} - \frac{2}{5}(1-x)^{5/2} + c$$

38) $\int_0^7 \sqrt{4+3x} \, dx = ?$

Let $u = 4+3x$ then $du = 3dx$ hence

$$\int_{x=0}^{x=7} \sqrt{4+3x} \, dx = \int_{x=0}^{x=7} \sqrt{u} . \frac{du}{3} = \left[\frac{2}{9}(4+3x)^{3/2}\right]_0^7 = \frac{2}{9}(125-8)$$

$$= \frac{2}{9} . (117) = 26$$

40) $\int_0^{\sqrt{\pi}} x.\cos(x^2)dx = ?$

Let $u=x^2$ then $du=2xdx$ hence

$$\int_{x=0}^{x=\sqrt{\pi}} x.\cos(x^2)dx = \int_{x=0}^{x=\sqrt{\pi}} \cos u.\frac{du}{2} = \left[\frac{\sin(x^2)}{2}\right]_{x=0}^{\sqrt{\pi}} = \frac{\sin|\pi|}{2} - \frac{\sin 0}{2}$$

$$= 0 - 0 = 0.$$

48) $\int_{x=0}^{x=\pi/2} \cos x . \sin(\sin x)\, dx = ?$

Let $u=\sin x$ then $du=\cos x dx$ hence

$$\int_{x=0}^{\pi/2} \cos x . \sin(\sin x)\, dx = \int_{x=0}^{\pi/2} \sin u\, du = \left[-\cos u\right]_{x=0}^{\pi/2} = \left[-\cos(\sin x)\right]_0^{\pi/2}$$

$$= -\cos\left(\sin\frac{\pi}{2}\right) + \cos(\sin 0)$$

$$= -\cos 1 + \cos 0$$

$$= 1 - \cos 1$$

58) Let $u=x^2$ then $du = 2xdx$ and for $x=0$, $u=0$, and for $x=1$, $u=1$ hence

$$A = \int_0^1 x\sqrt{1-x^4}\,dx = \int_0^1 \sqrt{1-u^2}\,\frac{du}{2} = \frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du \rightarrow$$ The area of a quarter-circle with radius 1 hence

$$A = \frac{1}{2}.\frac{1}{4}(\pi.1^2) = \frac{\pi}{8}$$

62) If f is continous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 x f(x^2)dx$.

Let $u = x^2$ then $du = 2x\,dx$ hence

$$\int_{x=0}^{x=3} x f(x^2)dx = \int_{u=0}^{u=9} f(u).\left(\tfrac{1}{2}du\right) = \frac{1}{2}.4 = 2.$$

66) Let $u = \pi - x$ then $du = -dx$

when $x = \pi$, $u = 0$ and when $x = 0$, $u = \pi$. So

$$\int_0^\pi x f(\sin x)dx = \int_0^\pi (\pi - u) f(\sin u)du$$

$$= \pi . \int_0^\pi f(\sin u)du - \int_0^\pi u . f(\sin u)\,du$$

$$= \pi . \int_0^\pi f(\sin x)\,dx - \int_0^\pi x . f(\sin x)\,dx$$

$$\Rightarrow 2\int_0^\pi x f(\sin x)dx = \pi \int_0^\pi f(\sin x)dx$$

$$\Rightarrow \int_0^\pi x f(\sin x)\,dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx.$$

section 6.1

2) $A = \int_{y=0}^{1} (x_R - x_L)dy = \int_0^1 [\sqrt{y} - (y^2 - 1)]dy = \int_0^1 (y^{\frac{1}{2}} - y^2 + 1)\,dy = \frac{4}{3}$

4) $A = \int_0^3 [(2y - y^2) - (y^2 - 4y)]dy = \int_0^3 (-2y^2 + 6y)dy = 9$

8) $A = \int_{-1}^{1} (x^2 - x^4)\,dx \xrightarrow[\text{even function}]{} = 2\int_0^1 (x^2 - x^4)\,dx = \frac{4}{15}$

12) $x = \sqrt[3]{x} \Rightarrow x^3 = x \Rightarrow x^3 - x = 0 \Rightarrow x(x^2-1) = 0 \Rightarrow x = -1, 0$ or 1

hence

$$A = \int_{-1}^{1} |\sqrt[3]{x} - x|\,dx = \int_{-1}^{0} (x - \sqrt[3]{x})\,dx + \int_0^1 (\sqrt[3]{x} - x)\,dx = 2\int_0^1 (x^{\frac{1}{3}} - x)\,dx$$
$$= \frac{1}{2}$$

22) $\sin x = \sin 2x = 2\sin x \cos x$ when $\sin x = 0$, $x = 0$, and $\cos x = \frac{1}{2}$,

$x = \frac{\pi}{3}$ hence

$$A = \int_0^{\pi/3} (\sin 2x - \sin x)\,dx + \int_{\pi/3}^{\pi/2} (\sin x - \sin 2x)\,dx$$

$$= \frac{1}{2}$$

24) For $x > 0$, $x = x^2 - 2 \Rightarrow 0 = x^2 - x - 2 \Rightarrow 0 = (x-2)(x+1)$
$\Rightarrow x = 2$

By symmetry, $\int_{-2}^{2} [|x| - (x^2-2)]\,dx = 2\int_0^2 [x - (x^2-2)]\,dx$

$$= \frac{20}{3}$$

45) By symmetry, we consider only the first quadrant where $y = x^2 \Rightarrow x = \sqrt{y}$. We're looking for a number b such that

$$\int_0^b \sqrt{y}\,dy = \int_b^4 \sqrt{y}\,dy \Rightarrow \left[\left[y^{3/2}\right]\cdot\frac{2}{3}\right]_0^b \Rightarrow b^{3/2} = 4^{3/2} - b^{3/2}$$

$$\Rightarrow 2b^{3/2} = 8$$

$$\Rightarrow b = 4^{2/3} \approx 2.52$$