HWK 11

pg 501

(26) $\lim_{x\to 0} \frac{\sin x - x}{x^3} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x - 1}{3x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin x}{6x}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{-\cos x}{6} = \boxed{\frac{-1}{6}}$

(28) $\lim_{x\to\infty} \frac{(\ln x)^2}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2\ln x\,(1/x)}{1} = \lim_{x\to\infty} \frac{2\ln x}{x}$

$\overset{L'H}{=} 2\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 2\lim_{x\to\infty} \frac{1}{x} = 2\cdot 0 = \boxed{0}$

(60) $\lim_{x\to 0} (\cos 3x)^{5/x}$ Let $y = (\cos 3x)^{5/x}$ then

$\lim_{x\to 0} y = \lim_{x\to 0} e^{\ln y} = e^{\lim_{x\to 0} \ln y}$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} 5/x \cdot \ln(\cos 3x) = 5\lim_{x\to 0} \frac{\ln(\cos 3x)}{x}$

$\overset{L'H}{=} 5\lim_{x\to 0} \frac{\frac{1}{\cos 3x}\cdot(-\sin 3x)\cdot 3}{1} = 5\lim_{x\to 0} \frac{-3\tan 3x}{1} = 0$

So $\lim_{x\to 0} (\cos 3x)^{5/x} = e^0 = \boxed{1}$

HWK 11 cont.

pg 516

(14) $\int t^3 e^t\,dt = t^3e^t - 3\int t^2 e^t\,dt$

$u = t^3 \quad dv = e^t dt$
$du = 3t^2 dt \quad v = e^t$

$u = 3t^2 \quad dv = e^t dt$
$du = 6t \quad v = e^t$

$= t^3e^t - (3t^2e^t - \int 6te^t\,dt)$
$= t^3e^t - 3t^2e^t + \int 6te^t\,dt$

$u = 6t \quad dv = e^t dt$
$du = 6\,dt \quad v = e^t$

$= t^3e^t - 3t^2e^t + 6te^t - \int 6e^t\,dt$
$= t^3e^t - 3t^2e^t + 6te^t - 6e^t + C$
$= e^t(t^3 - 3t^2 + 6t - 6) + C$

(22) $\int_1^2 \frac{\ln x}{x^2}\,dx = \frac{-\ln x}{x}\Big|_1^2 + \int_1^2 \frac{1}{x^2}\,dx$

$\boxed{\begin{array}{ll} u = \ln x & dv = \frac{1}{x^2}\,dx \\ du = \frac{1}{x}\,dx & v = -\frac{1}{x} \end{array}}$

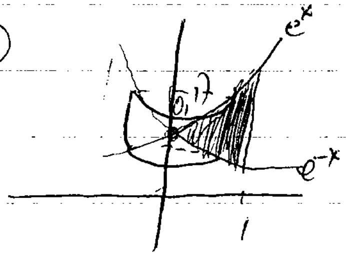
$= \frac{-\ln x}{x}\Big|_1^2 - \frac{1}{x}\Big|_1^2$

$= \frac{-\ln 2}{2} + \frac{\ln 1}{1}^{\,0} - \frac{1}{2} + \frac{1}{1}$

$= \frac{1}{2} - \frac{1}{2}\ln 2$

HWK 11 cont.

(56)



$$V = 2\pi \int_0^1 x\,(e^x - e^{-x})\,dx$$

$$= 2\pi\left[\int_0^1 x e^x\,dx - \int_0^1 x e^{-x}\,dx\right]$$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$u = x \quad dv = e^{-x} dx$

$du = dx \quad v = -e^{-x}$

$r = x$

$h = e^x - e^{-x}$

$$= 2\pi\left[x e^x\Big|_0^1 - \int_0^1 e^x\,dx - \left[-x e^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right]\right]$$

$$= 2\pi\left[x e^x\Big|_0^1 - e^x\Big|_0^1 + x e^{-x}\Big|_0^1 + e^{-x}\Big|_0^1\right]$$

$$= 2\pi\left[(1\cdot e^1 - 0) - (e^1 - e^0) + (1e^{-1} - 0) + (e^{-1} - e^0)\right]$$

$$= 2\pi\left[e - e + 1 + \frac{1}{e} + \frac{1}{e} - 1\right]$$

$$= 2\pi\left[\frac{2}{e}\right]$$

$$= 4\pi/e$$