Homework set # 7 (solution of graded problems)

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26.

$$f(x) = \frac{4x-1}{2x+3} \qquad f^{-1}(x) = ?$$

Let $f(x) = y$,

$$y = \frac{4x-1}{2x+3} \Rightarrow y(2x+3) = 4x-1$$

$$\Rightarrow 2xy + 3y = 4x - 1$$

$$\Rightarrow 2xy - 4x = -1 - 3y$$

$$\Rightarrow x(2y-4) = -1-3y.$$

$$\therefore x = \frac{3y+1}{4-2y}$$

$$\therefore f^{-1}(x) = \frac{3x+1}{4-2x} \quad (\text{interchanging } x \text{ & } y)$$

30. $f(x) = 2x^2 - 8x \quad x \geq 2. \qquad f^{-1}(x) = ?$

Let $y = f(x)$,

$$y = 2x^2 - 8x \qquad x \geq 2$$

$$\Rightarrow 2x^2 - 8x - y\,0 \qquad x \geq 2$$

$$\Rightarrow x = \frac{8 \pm \sqrt{64 + 8y}}{4} \quad \underline{x \geq 2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2$
$b = -8$
$c = -y$.

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$\Rightarrow x = 2 + \frac{1}{2}\sqrt{16+2y}$

Interchanging x and y,

$$\boxed{f^{-1}(x) = 2 + \frac{1}{2}\sqrt{16+2x}}$$

#40.

Find $(f^{-1})'(a)$.

$f(x) = x^5 - x^3 + 2x \quad a = 2.$

$f(1) = 2$

$\Rightarrow f^{-1}(2) = 1 \quad \& \quad f(x) = x^5 - x^3 + 2x$

$f'(x) = 5x^4 - 3x^2 + 2$

$f'(1) = 4$

From thm 7 $\quad (f^{-1})'(2) = \dfrac{1}{f'(f^{-1}(2))}$

$= \dfrac{1}{f'(1)}$

$= \boxed{\dfrac{1}{4}}$

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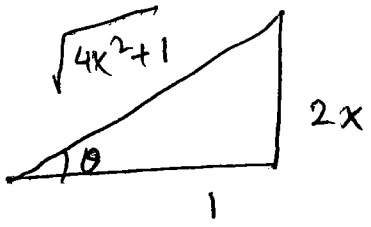
14. $\csc(\arctan 2x)$

Let $\theta = \arctan 2x$

$\Rightarrow \tan\theta = 2x$



$$\csc\theta = \frac{\sqrt{4x^2+1}}{2x}$$

$$\Rightarrow \boxed{\csc(\arctan 2x) = \frac{\sqrt{4x^2+1}}{2x}}$$

62 $\int \frac{dt}{\sqrt{1-4t^2}}$

Put, $u = 2t$

$du = 2\,dt$.

$$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u + C$$

$$= \boxed{\frac{1}{2}\sin^{-1}(2t) + C.}$$

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$$\int \frac{\tan^{-1}x}{1+x^2}\,dx.$$

$u = \tan^{-1}x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$\Rightarrow du = \frac{dx}{1+x^2}$

$$\Rightarrow \int u\,du = \frac{u^2}{2} + C$$

$$= \boxed{\frac{1}{2}\left[\tan^{-1}x\right]^2 + C}$$