Lamoureux/Orjiako

Fifteen puzzle

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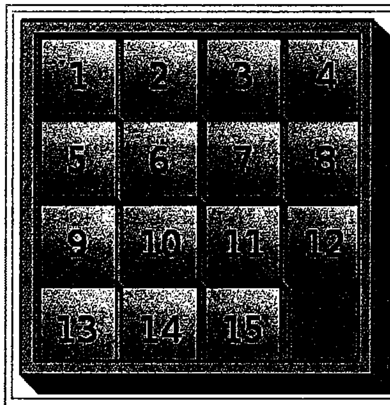


A solved 15-puzzle

The ***n*-puzzle** is known in various versions, including the **8 puzzle**, the **15 puzzle**, and with various names (**Gem Puzzle, Boss Puzzle, Game of Fifteen, Mystic Square** and many others). It is a sliding puzzle that consists of a frame of numbered square tiles in random order with one tile missing. If the size is 3×3, the puzzle is called the 8-puzzle or 9-puzzle, and if 4×4, the puzzle is called the 15-puzzle or 16-puzzle. The object of the puzzle is to place the tiles in order (see diagram) by making sliding moves that use the empty space.

The n-puzzle is a classical problem for modelling algorithms involving heuristics. Commonly used heuristics for this problem include counting the number of misplaced tiles and finding the sum of the Manhattan distances between each block and its position in the goal configuration. Note that both are *admissible*, i.e., they never overestimate the number of moves left, which ensures optimality for certain search algorithms such as A*.

Contents

Solvability

Johnson & Story (1879) used a parity argument to show that half of the starting positions for the *n*-puzzle are impossible to resolve, no matter how many moves are made. This is done by considering a function of the tile configuration that is invariant under any valid move, and then using this to partition the space of all possible labeled states into two equivalence classes of reachable and unreachable states.

The invariant is the parity of permutations of all 16 squares (15 pieces plus empty square) plus the parity of the taxicab distance moved by the empty square. This is an invariant because each move changes the parity of the permutation and the parity of the taxicab distance. In particular if the empty square is not moved the permutation of the remaining pieces must be even.

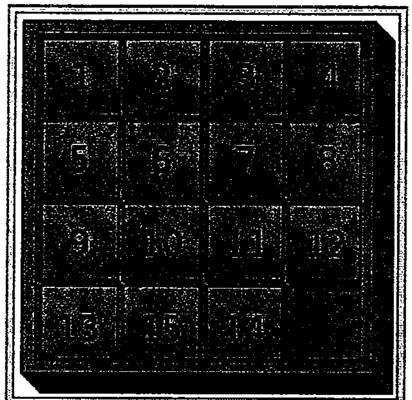
Johnson & Story (1879) also showed that the converse holds on boards of size $m \times n$ provided m and n are both at least 2: all even permutations *are* solvable. This is straightforward but a little messy to prove by induction on m and n starting with $m=n=2$. Archer (1999) gave another proof, based on defining equivalence classes via a hamiltonian path.

Wilson (1974) studied the analogue of the 15 puzzle on arbitrary finite connected and non-separable graphs. (A graph is called separable if removing a vertex increases the number of components.) He showed that, except for one exceptional graph on 7 vertices, it is possible to obtain all permutations unless the graph is bipartite, in which case exactly the even permutations can be obtained. The exceptional graph is a regular hexagon with one diagonal and a vertex at the center added.

For larger versions of the *n*-puzzle, finding a solution is easy, but the problem of finding the *shortest* solution is NP-hard.[1] For the 15-puzzle, lengths of optimal solutions range from 0 to 80 moves; the 8-puzzle can be solved in 31 moves or fewer (integer sequence A087725 (http://www.research.att.com/~njas/sequences/A087725)).

History

Sam Loyd claimed from 1891 until his death in 1911 that he invented the puzzle. But he had nothing to do with the invention or popularity of the puzzle.[2] The puzzle was "invented" by Noyes Palmer Chapman, a postmaster in Canastota, New York, who is said to have shown friends, as early as 1874, a precursor puzzle consisting of 16 numbered blocks that were to be put together in rows of four, each summing to 34. Copies of the improved Fifteen Puzzle made their way to Syracuse, New York by way of Noyes' son, Frank, and from there, via sundry connections, to Watch Hill, RI, and finally to Hartford (Connecticut), where students in the American School for the Deaf started manufacturing the puzzle and, by December 1879, selling them both locally and in Boston (Massachusetts). Shown one of these, Matthias Rice, who ran a fancy woodworking business in Boston, started manufacturing the puzzle sometime in December 1879 and convinced a "Yankee Notions" fancy goods dealer to sell them under the name of "Gem Puzzle". In late-January 1880, Dr. Charles Pevey, a dentist in Worcester, Massachusetts, garnered some attention by offering a cash reward for a solution to the Fifteen Puzzle.[2]



Sam Loyd's unsolvable 15-puzzle, with tiles 14 and 15 exchanged. This puzzle is not solvable because it would require a change of the invariant.

The game became a craze in the U.S. in February 1880, Canada in March, Europe in April, but that craze had pretty much dissipated by July. Apparently the puzzle was not introduced to Japan until 1889. The craze was, in part, fuelled by Loyd offering a $1,000 prize for anyone who could provide a solution for achieving a particular combination specified by Loyd.[3] Johnson & Story (1879) showed that no solution to this "14-15 puzzle" was possible as it required a transformation from an even to an odd combination. Noyes Chapman had applied for a patent on his "Block Solitaire Puzzle" on February 21, 1880. However, that patent was rejected, likely because it was not sufficiently different from the August 20, 1878 "Puzzle-Blocks" patent (US 207124) granted to Ernest U. Kinsey.[2]

Miscellaneous

The Minus Cube, manufactured in the USSR, is a 3D puzzle with similar operations to the 15-puzzle.

An electronic 3x3x3 three dimensional n-puzzle can be downloaded from [1] (http://www.v3minus.com)

Bobby Fischer was an expert at solving the 15-Puzzle. He had been timed to be able to solve it within 25 seconds; Fischer demonstrated this on November 8, 1972 on *The Tonight Show Starring Johnny Carson.*

See also

- Rubik's Cube
- Minus Cube
- Sliding puzzle
- Combination puzzles
- Mechanical puzzles
- Jeu de taquin, an operation on skew Young tableaux similar to moves of the 15 puzzle.

Notes

1. ^ Daniel Ratner, Manfred K. Warmuth. *Finding a Shortest Solution for the N x N Extension of the 15-PUZZLE Is Intractable.*

National Conference on Artificial Intelligence, 1986.
2. ^ [a] [b] [c] *The 15 Puzzle*, by Jerry Slocum & Dic Sonneveld. ISBN 1-890980-15-3
3. ^ Richard E. Korf, "Recent progress in the design and analysis of admissible heuristic functions (http://books.google.co.uk/books?id=09NVbhIlJ-UC&pg=PA45&dq=%22design+and+analysis+of+admissible+heuristic+functions%22+isbn:3540678395&lr=&as_brr=0#v=onepage&q=%22design%20and%20analysis%20of%20admissible%20heuristic%20functions%22%20isbn%3A3540678395&f=false) ", *Abstraction, reformulation, and approximation: 4th international symposium*, Texas, USA, 26-29 July 2000, pp.45-57, Springer, 2000 ISBN 3540678395.

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