PROBLEM: Find equation of tangent plane to $z = f(x,y)$ at the point $(a,b,f(a,b))$.

① The vertical plane $x=a$ cuts the graph in a curve C:

$$\vec{r}(y) = \langle a, y, f(a,y) \rangle$$

② $\dfrac{d\vec{r}}{dy}\Big|_{y=b} = \langle 0, 1, f_y(a,b) \rangle$

is a tangent vector to C at $(a,b,f(a,b))$.

① The vertical plane $y=b$ cuts the graph in a curve D:

$$\vec{r}(x) = \langle x, b, f(x,b) \rangle$$

② $\dfrac{d\vec{r}}{dx}\Big|_{x=a} = \langle 1, 0, f_x(a,b) \rangle$

is a tangent vector to D at $(a,b,f(a,b))$.

③ $\vec{N} = \langle 1, 0, f_x(a,b) \rangle \times \langle 0, 1, f_y(a,b) \rangle$

$$= \langle -f_x(a,b), -f_y(a,b), 1 \rangle$$

is a normal vector to the tangent plane.

④ Equation of tangent plane is

$$(-f_x(a,b))(x-a) + (-f_y(a,b))(y-b) + (1)(z-f(a,b)) = 0$$

OR

TANGENT PLANE

$$\boxed{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$