The hard part of this problem is determining which series to compare $\sum(1 - \cos(1/n))$ with. Recall that we could compare (via Limit Form of Comparison Test) the series $\sum \sin(1/n)$ with the harmonic series $\sum 1/n$. This was because $\sin x$ behaves like x for small values of x. We usually state this result as a limit:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Taking a look at the graph of $\sin x$ we see that it does indeed look like the line $y = x$ near the origin.

But what about the graph of $y = 1 - \cos(x)$? This is just the cosine graph flipped upside down and shifted up one unit (**draw this!**). You might hazard a guess that it looks a bit like a parabola near the origin. Let's test this guess by computing the limit of $(1 - \cos(x))/(x^2)$ as $x \to 0$.

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{\sin x}{2x} = \frac{1}{2}$$

The first equality comes from L'Hopital's rule, and the second inequality comes from the sine limit above.

What this means is that we can compare $\sum(1 - \cos(1/n))$ with the convergent p-series $\sum \frac{1}{n^2}$. From the previous paragraph, we know that $\lim a_n/b_n = 1/2$, and so the limit form of the comparison test tells us that our series is convergent.