

Calculus III [2433–001] Quiz I

Wednesday, January 27, 1999

SOLUTIONS

Q1]... Determine if the following series is convergent. If it is, determine its sum.

$$\sum_{n=1}^{\infty} \left[\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right) \right]$$

Solution: The n th partial sum is just

$$s_n = \left[\cos\left(\frac{1}{1}\right) - \cos\left(\frac{1}{2}\right) \right] + \cdots + \left[\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right) \right]$$

which is a telescoping sum, and reduces down to

$$s_n = \cos\left(\frac{1}{1}\right) - \cos\left(\frac{1}{n+1}\right).$$

Taking $\lim_{n \rightarrow \infty}$ gives us a limit of

$$s = \cos(1) - \cos(0) = \cos(1) - 1.$$

We have shown that the series is convergent, with sum $s = \cos(1) - 1$.