Math 240: Elementary Differential Equations

Final Exam: Summer 2008

August 1, 2008

Name.....

Instructor.....

This is a closed book exam. You can use a calculator and 8.5" $\times$ 11" sheet of hand written note (both sides).Detail work must be shown for the full credit.

(10)Problem 1. Solve the initial value problem

$$\frac{dy}{dx} = x^2y + 4y, \quad y(0) = 1.$$

(12)Problem 2. The population P of fish in a lake is governed by the differential equation

$$\frac{dP}{dt} = -P^3 + 10P^2 - 21P.$$

(a) Find the equilibrium points and classify them by sketching several trajectories.Also indicate threshold and carrying capacity.

(b) If P represents the population in thousands then what should be the minimum population of the fish in the lake in order to prevent the extinction ?

(12) Problem 3. A mass of $30g$ is attached to a spring, which causes spring to stretch $20cm$. The damping constant of the spring is $1kg/sec$. The mass is subject to an external force of $4\cos(5t)$ Newtons. Find the equation of the resulting motion.

(12)Problem 4. Solve the system of equations

$$\frac{dx}{dt} = 2x - 4y, \quad x(0) = 0$$

$$\frac{dy}{dt} = 3x - 5y, \quad y(0) = 1.$$

(12)Problem 5. Solve the initial value problem

$$x'' + 4x' + 3x = \delta(t-1), \quad x(0) = 0, x'(0) = 0.$$

(12)Problem 6. Find and classify the equilibrium points of the following system of equations as stable, unstable or unable to tell.

$$\frac{dx}{dt} = x + y, \qquad \frac{dy}{dt} = x^2 + y^2 - 2.$$

(12) Problem 7.

(a) Express the function $\frac{x}{1+x}$ as Taylor series about $x_0 = 0$ and mention its radius of convergence.

(b) Suppose the differential equation $(x^2 - x - 6)(x^2 + 1)y'' - (3x - 2)y' + (x^2 + 4x + 1) = 0$ is solved as a power series about $x_0 = 2$. Find the lower bound for the radius of convergence of the series solution.

(c) Find and classify the singular points of the equation $(x^3 + x)y'' + 3y' + xy = 0$.

(12) Problem 8. Find the series solution of the initial value problem

$$(x+1)y'' - y = 0, \quad y(0) = -1, y'(0) = 1.$$

(10)Problem 9. Solve the Euler equation

$$x^2y'' - 5xy' + 9y = 0.$$

(16)Problem 10. Consider the differential equation $x^2y'' + (x^2 - x)y' + y = 0$.

(a) Find the roots of the indicial equation corresponding to the regular singular point $x_0 = 0$.

(b) Find the series solution about $x_0 = 0$ corresponding to the larger root of the indicial equation.